Foundations of Artificial Intelligence

Prof. Dr. W. Burgard, Prof. Dr. B. Nebel, Prof. Dr. M. Riedmiller
J. Aldinger, B. Frank
University of Freiburg
Department of Computer Science
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Exercise Sheet 4

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Exercise 4.1 (Satisfiability and Resolution)

(a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.
 (i) $\mathit{Smoke} \Rightarrow \mathit{Smoke}$
 (ii) $\mathit{Smoke} \Rightarrow \mathit{Fire}$
 (iii) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow (\neg \mathit{Fire} \Rightarrow \neg \mathit{Smoke})$
 (iv) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow ((\mathit{Smoke} \wedge \mathit{Heat}) \Rightarrow \mathit{Fire})$
 (v) $\mathit{TheBestTeamWins} \Leftrightarrow \mathit{GermanyWinsTheEuro2012}$

(b) Let $K = \{\{A, B, \neg C\}, \{\neg A, C\}, \{\neg A, \neg B\}, \{A, C\}\}$. Use the resolution method to show that $K \models (\neg B \Rightarrow (A \wedge C))$.

Exercise 4.2 (Davis-Putnam Procedure)

Use the Davis-Putnam procedure to compute models for the following clause sets or to prove that no model exists. Whenever possible, apply *unit propagation*. At each step, indicate which rule you have applied.

(a) $\big\{\{P, \neg Q\}, \{\neg P, Q\}, \{Q, \neg R\}, \{S\}, \{\neg S, \neg Q, \neg R\}, \{S, R\}\big\}$
(b) $\big\{\{P, Q, S, T\}, \{P, S, \neg T\}, \{Q, \neg S, T\}, \{P, \neg S, \neg T\}, \{P, \neg Q\}, \{\neg R, \neg P\}, \{R\}\big\}$

Exercise 4.3

Consider following colloquial sentences:

(a) Not all students attend AI and ST.

(b) One student failed both AI and ST.

(c) Exactly two students failed ST.

(d) There is a barber who shaves all men in town who do not shave themselves.

(e) No one likes a professor who is not smart.

Represent these sentences in first-order logic using the predicates *student(x)*, *attends(x,y)*, *fails(x,y)*, *barber(x)*, *shaves(x,y)*, *professor(x)*, *likes(x,y)* und *smart(x)*.

Exercise 4.4

Consider the following set of formulae Θ and the interpretation $\mathcal{I}$:

- $\Theta = \{\texttt{Person(a)}, \texttt{Person(b)}, \forall \texttt{x}\ (\texttt{Person(x)} \Rightarrow (\texttt{Small(x)} \vee \texttt{Stupid(x)}))$
- $\mathrm{D} = \{d_1, d_2, d_3\}$
- $\texttt{a}^{\mathcal{I}} = d_1, \texttt{b}^{\mathcal{I}} = d_2$

- $\texttt{Person}^{\mathcal{I}} = \{d_1, d_2, d_3\}$
- $\texttt{Small}^{\mathcal{I}} = \{d_1\}, \texttt{Stupid}^{\mathcal{I}} = \{d_3\}$
- $\alpha = \{(\texttt{x} \mapsto d_1), (\texttt{y} \mapsto d_2)\}$

Answer the following questions, i.e. state whether interpretation $\mathcal{I}$ under α is a model of the respective formula or not. Explain your answers.

(a) $\mathcal{I}, \alpha \models \forall \texttt{x}\ (\texttt{Person(x)} \Rightarrow (\texttt{Small(x)} \vee \texttt{Stupid(x)})$?

(b) $\mathcal{I}, \alpha \models \texttt{Person(x)} \Rightarrow (\texttt{Small(x)} \vee \texttt{Stupid(x)})$?

(c) $\mathcal{I}, \alpha \models \texttt{Small(y)}$?

(d) $\mathcal{I}, \alpha \models \exists \texttt{y}\ \texttt{Stupid(y)}$?

(e) $\mathcal{I}, \alpha \models \Theta$?

Exercise 4.5

(a) Transform the following formula into Skolem Normal Form (SNF):

$$\forall z \exists y (P(x, g(y), z) \vee \neg \forall x Q(x)) \wedge \neg \forall z \exists x \forall t \neg R(f(x, z), z, t)$$

(b) Give the 10 smallest terms in the Herbrand universe and the 10 smallest formulae in the Herbrand expansion of the following formula:

$$\forall x \forall y P(c, f(x, b), g(y))$$

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.