Foundations of Artificial Intelligence 11. Action Planning Solving Logically Specified Problems using a General Problem Solver

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Contents



- Planning Formalisms
- Basic Planning Algorithms
- Computational Complexity
- 5 Current Algorithmic Approaches
- Summary

- Planning is the process of generating (possibly partial) representations of future behavior prior to the use of such plans to constrain or control that behavior.
- The outcome is usually a set of actions, with temporal and other constraints on them, for execution by some agent or agents.
- As a core aspect of human intelligence, planning has been studied since the earliest days of AI and cognitive science. Planning research has led to many useful tools for real-world applications, and has yielded significant insights into the organization of behavior and the nature of reasoning about actions. [Tate 1999]

Given a current state, a set of possible actions, a specification of the goal conditions, which plan transforms the current state into a goal state?



Given a road map, and a number of trucks and airplanes, make a plan to transport objects from their start to their goal destinations.



- Problem solving by search, where we describe a problem by a state space and then implement a program to search through this space
 - in action planning, we specify the problem declaratively (using logic) and then solve it by a general planning algorithm
- Program synthesis, where we generate programs from specifications or examples
 - in action planning we want to solve just one instance and we have only very simple action composition (i.e., sequencing, perhaps conditional and iteration)
- Scheduling, where all jobs are known in advance and we only have to fix time intervals and machines
 - instead we have to find the right actions and to sequence them
- Of course, there is interaction with these areas!

- Start with a declarative specification of the planning problem
- Use a domain-independent planning system to solve the planning problem
- Domain-independent planners are generic problem solvers
- Issues:
 - Good for evolving systems and those where performance is not critical
 - Running time should be comparable to specialized solvers
 - Solution quality should be acceptable
 - ... at least for all the problems we care about

Planning can be elegantly formalized with the help of the *situation calculus*.

Initial state:

 $At(truck1, loc1, s_0) \land At(package1, loc3, s_0)$

Operators (successor-state axioms): $\forall a, s, l, p, t \ At(t, p, Do(a, s)) \Leftrightarrow \{a = Drive(t, l, p) \land Poss(Drive(t, l, p), s) \land (a \neq \neg Drive(t, p, l, s) \lor \neg Poss(Drive(t, p, l), s))\}$

Goal conditions (query):

 $\exists s \ At(package1, loc2, s)$

The constructive proof of the existential query (computed by a automatic theorem prover) delivers a plan that does what is desired. Can be quite inefficient!

STRIPS: STanford Research Institute Problem Solver

- S is a *first-order vocabulary* (predicate and function symbols) and Σ_S denotes the set of *ground atoms* over the signature (also called facts or fluents).
- $\Sigma_{S,V}$ is the set of atoms over S using variable symbols from the set of variables V.
- A first-order STRIPS state S is a subset of Σ_S denoting a complete theory or model (using CWA).
- A planning task (or planning instance) is a 4-tuple $\Pi = \langle S, \mathbf{O}, \mathbf{I}, \mathbf{G} \rangle$, where
 - O is a set of operator (or action types)
 - $I \subseteq \Sigma_S$ is the initial state
 - $\mathbf{G} \subseteq \Sigma_{\mathcal{S}}$ is the goal specification
- No domain constraints (although present in original formalism)

Operators, Actions & State Change

• Operator:

$$o = \langle para, pre, eff \rangle$$
,

with $para \subseteq V$, $pre \subseteq \Sigma_{S,V}$, $eff \subseteq \Sigma_{S,V} \cup \neg \Sigma_{S,V}$ (element-wise negation) and all variables in *pre* and *eff* are listed in *para*. Also: pre(o), eff(o). $eff^+ = positive effect literals$ $eff^- = negative effect literals$

- Operator instance or action: Operator with empty parameter list (instantiated schema!)
- State change induced by action:

$$App(S,o) = \begin{cases} S \cup eff^+(o) - \neg eff^-(o) & \text{if } pre(o) \subseteq S \& \\ eff(o) \text{ is cons.} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Example Formalization: Logistics

- Logical atoms: at(O,L), in(O,V), airconn(L1,L2), street(L1,L2), plane(V), truck(V)
- Load into truck: *load* Parameter list: (O, V, L)Precondition: at(O, L), at(V, L), truck(V)Effects: $\neg at(O, L), in(O, V)$

• Drive operation: *drive* Parameter list: (V, L1, L2)Precondition: at(V, L1), truck(V), street(L1, L2)Effects: $\neg at(V, L1), at(V, L2)$

• . . .

- Some constant symbols: v1, s, t with truck(v1) and street(s, t)
- Action: drive(v1, s, t)

- A plan Δ is a sequence of actions
- State resulting from executing a plan:

$$Res(S, \langle \rangle) = S$$

$$Res(S, (o; \Delta)) = \begin{cases} Res(App(S, o), \Delta) & \text{if } App(S, o) \\ & \text{is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

• Plan Δ is successful or solves a planning task if $Res(\mathbf{I}, \Delta)$ is defined and $\mathbf{G} \subseteq Res(\mathbf{I}, \Delta)$.

Initial state:
$$S = \left\{ \begin{array}{l} at(p1,c), at(p2,s), at(t1,c), \\ at(t2,c), street(c,s), street(s,c) \end{array} \right\}$$

Goal: G =
$$\{ at(p1, s), at(p2, c) \}$$

Successful plan:
$$\Delta = \langle load(p1,t1,c), drive(t1,c,s), unload(p1,t1,s), load(p2,t1,s), drive(t1,s,c), unload(p2,t1,c) \rangle$$

Other successful plans are, of course, possible

Simplifications: DATALOG- and Propositional STRIPS

- STRIPS as described above allows for unrestricted first-order terms, i.e., arbitrarily nested function terms
- \rightarrow Infinite state space
 - Simplification: No function terms (only 0-ary = constants)
- \rightarrow **DATALOG-STRIPS**
 - Simplification: No variables in operators (= actions)
- → Propositional STRIPS
 - Propositional STRIPS used in planning algorithms nowadays (but specification is done using DATALOG-STRIPS)

Beyond STRIPS

Even when keeping all the restrictions of classical planning, one can think of a number of extensions of the planning language.

- General logical formulas as preconditions: Allow all Boolean connectors and quantification
- Conditional effects: Effects that happen only if some additional conditions are true. For example, when pressing the accelerator pedal, the effects depends on which gear has been selected (no, reverse, forward).
- Multi-valued state variables: Instead of 2-valued Boolean variables, multi-valued variables could be used
- Numerical resources: Resources (such as fuel or time) can be effected and be used in preconditions
- Durative actions: Actions can have duration and can be executed concurrently
- Axioms/Constraints: The domain is not only described by operators, but also by additional laws

- Since 1998, there exists a bi-annual scientific competition for action planning systems.
- In order to have a common language for this competition, PDDL has been created (originally by Drew McDermott)
- Meanwhile, version 3.2 (IPC-2011) with most of the features mentioned.
- Sort of standard language by now.

(define (domain logistics) (:types truck airplane - vehicle package vehicle - physobj airport location - place city place physobj - object)

```
(:predicates (in-city ?loc - place ?city - city)
(at ?obj - physobj ?loc - place)
(in ?pkg - package ?veh - vehicle))
```

(:action LOAD-TRUCK

:parameters (?pkg - package ?truck - truck ?loc - place) :precondition (and (at ?truck ?loc) (at ?pkg ?loc)) :effect (and (not (at ?pkg ?loc)) (in ?pkg ?truck))) ...)

Planning Problems as Transition Systems

- We can view planning problems as searching for goal nodes in a large labeled graph (transition system)
- Nodes are defined by the value assignment to the fluents = states
- Labeled edges are defined by actions that change the appropriate fluents
- Use graph search techniques to find a (shortest) path in this graph!
- Note: The graph can become huge: 50 Boolean variables lead to 2^{50} = 10^{15} states
- $\rightarrow\,$ Create the transition system on the fly and visit only the parts that are *necessary*

Transition System: Searching Through the State Space



Search through transition system starting at initial state

- Initialize partial plan Δ := () and start at the unique initial state I and make it the current state S
- 2 Test whether we have reached a goal state already: G ⊆ S? If so, return plan Δ.
- Select one applicable action o_i non-deterministically and
 - compute successor state $S := App(S, o_i)$,
 - extend plan $\Delta := \langle \Delta, o_i \rangle$, and continue with step 2.

Instead of non-deterministic choice use some search strategy. Progression planning can be easily extended to more expressive planning languages

Progression Planning: Example



Regression Planning: Backward Search

Search through transition system starting at goal states. Consider sets of states, which are described by the atoms that are necessarily true in them

 $lacksymbol{0}$ Initialize partial plan $\Delta:=\langle\
angle$ and set $\mathbf{S}:=\mathbf{G}$

2 Test whether we have reached the unique initial state already: $I \supseteq S$? If so, return plan Δ .

- Select one action o_i non-deterministically which does not make (sub-)goals false (S $\cap \neg eff^-(o_i) = \emptyset$) and
 - compute the regression of the description S through o_i :

$$\mathbf{S} := \mathbf{S} - eff^+(o_i) \cup pre(o_i)$$

• extend plan $\Delta := \langle o_i, \Delta \rangle$, and continue with step 2.

Instead of non-deterministic choice use some search strategy Regression becomes much more complicated, if e.g. conditional effects are allowed. Then the result of a regression can be a general Boolean formula

Regression Planning: Example



- Of course, other types of search are possible.
- Change perspective: Do not consider the transition system as the space we have to explore, but consider the search through the space of (incomplete) plans:
 - Progression search: Search through the space of plan prefixes
 - Regression search: Search through plan suffixes
- Partial order planning:
 - Search through partially ordered plans by starting with the empty plan and trying to satisfy (sub-)goals by introducing new actions (or using old ones)
 - Make ordering choices only when necessary to resolve conflicts

Definition (Plan existence problem (PLANEX))

Instance: $\Pi = \langle S, \mathbf{O}, \mathbf{I}, \mathbf{G} \rangle$.

Question: Does there exist a plan Δ that solves Π , i.e., $Res(I, \Delta) \supseteq G$?

Definition (Bounded plan existence problem (PLANLEN))

Instance: $\Pi = \langle S, \mathbf{O}, \mathbf{I}, \mathbf{G} \rangle$ and a positive integer *n*. Question: Does there exist a plan Δ of length *n* or less that solves Π ?

From a practical point of view, also PLANGEN (*generating* a plan that solves II) and PLANLENGEN (*generating* a plan of length n that solves II) and PLANOPT (generating an optimal plan) are interesting (but at least as hard as the decision problems).

Basic STRIPS with First-Order Terms

- The state space for STRIPS with general first-order terms is infinite
- We can use function terms to describe (the index of) tape cells of a Turing machine
- We can use operators to describe the Turing machine control
- The existence of a plan is then equivalent to the existence of a successful computation on the Turing machine
- PLANEX for STRIPS with first-order terms can be used to decide the Halting problem

Theorem

PLANEX for STRIPS with first-order terms is undecidable.

Theorem

PLANEX is **PSPACE-complete** for propositional STRIPS.

- \rightarrow Membership follows because we can successively guess operators and compute the resulting states (needs only polynomial space)
- → Hardness follows using again a generic reduction from TM acceptance. Instantiate polynomially many tape cells with no possibility to extend the tape (only poly. space, can all be generated in poly. time)
 - PLANLEN is also PSPACE-complete (membership is easy, hardness follows by setting k = 2^{|Σ|})

- If we restrict the length of the plans to be only polynomial in the size of the planning task, PLANEX becomes NP-complete
- Similarly, if we use a unary representation of the natural number *k*, then PLANLEN becomes NP-complete
- \rightarrow Membership obvious (guess & check)
- $\rightarrow\,$ Hardness by a straightforward reduction from SAT or by a generic reduction.
 - One source of complexity in planning stems from the fact that plans can become very long
 - We are only interested in short plans!
 - We can use methods for NP-complete problems if we are only looking for "short" plans.

Propositional, Precondition-free STRIPS with Negative Preconditions

Theorem

The problem of deciding plan existence for precondition-free, propositional STRIPSis in P.

Proof.

Do a backward greedy plan generation. Choose all operators that make some goals true and that do not make any goals false. Remove the satisfied goals and the operators from further consideration and iterate the step. Continue until all remaining goals are satisfied by the initial state (succeed) or no more operators can be applied (fail).

Propositional, Precondition-free STRIPS and Plan Length

Theorem

The problem of deciding whether there exists a plan of length *k* for precondition-free, propositional STRIPSis NP-complete, even if all effects are positive.

Proof.

Membership in NP is obvious. Hardness follows from a straightforward reduction from the MINIMUM-COVER problem [Garey & Johnson 79]:

Given a collection *C* of subsets of a finite set *S* and a positive integer *k*, does there exist a cover for *S* of size *k* or less, i.e., a subset $C' \subseteq C$ such that $\bigcup C' \supseteq S$ and $|C'| \le k$?

We will use this result later

- In 1992, Kautz and Selman introduced the idea of planning as satisfiability
- \rightarrow Encode possible *k*-step plans as Boolean formulas and use an iterative deepening search approach
 - In 1995, Blum and Furst came up with the planning graph approach
- $\rightarrow\,$ iterative deepening approach that prunes the search space using a graph-structure
 - In 1996, McDermott proposed to use (again) an heuristic estimator to control the selection of actions, similar to the original GPS idea
 - Geffner (1997) followed up with a propositional, simplified version (HSP) and Hoffmann & Nebel (2001) with an extended version integrating strong pruning (FF)
- $\rightarrow\,$ Heuristic planners seem to be the most efficient non-optimal planners these days

Initialize k = 0

- Iry to construct a plan of length k exhaustively
- If unsuccessful, increment k and goto step 2.
- Otherwise return plan
- Finds shortest plan
- Needs to prove that there are no plans of length 1, 2, ... k − 1 before a plan of length k is produced.

- Traditionally, planning has been viewed as a special kind of deductive problem
- Given
 - a formula describing possible state changes
 - a formula describing the initial state and a formula characterizing the goal conditions
 - try to prove the existential formula there exists a sequence of state changes transforming the initial state into the final one
 - $\rightarrow\,$ Since the proof is done constructively, the plan is constructed as a by-product

- Take the dual perspective: Consider all models satisfying a particular formula as plans
- → Similar to what is done in the generic reduction that shows NP-hardness of SAT (simulation of a computation on a Turing machine)
 - Build formula for *k* steps, check satisfiability, and increase *k* until a satisfying assignment is found
 - Use time-indexed propositional atoms for facts and action occurrences
 - Formulate constraints that describe what it means that a plan is successfully executed:
 - Only one action per step
 - If an action is executed then their preconditions were true and the effects become true after the execution
 - If a fact is not affected by an action, it does not change its value (frame axiom)

Planning as Satisfiability: Example

- Fact atoms: $at(p1,s)_i, at(p1,c)_i, at(t1,s)_i, at(t1,c)_i, in(p1,t1)_i$
- Action atoms: $move(t1, s, c)_i, move(t1, c, s)_i, load(p1, s)_i, \ldots$
- Only one action: $\bigwedge_{i,x,y} \neg (unload(t1,p1,x)_i \land load(p1,t1,y)_i) \land \ldots$
- Preconditions: $\bigwedge_{i,x}(unload(p1,t1,x)_i \rightarrow in(p1,t1)_{i-1}) \land \dots$
- Effects: $\bigwedge_{i,x}(unload(p1,t1,x)_i \rightarrow \neg in(p1,t1)_i \land at(p1,x)_i) \land \dots$
- Frame axioms:
 - $\bigwedge_{i,x,y,z} (\neg move(t1,x,y)_i \to (at(t1,z)_{i-1} \leftrightarrow at(t1,z)_i)) \land \dots$
- A satisfying truth assignment corresponds to a plan (use the true action atoms)

- Has a more flexible search strategy
- Can make use of SAT solver technology
- ... and automatically profits from advances in this area
- Can express constraints on intermediate states
- Can use logical axioms to express additional constraints, e.g., to prune the search space

Main ideas:

- Describe *possible* developments in a graph structure (use only positive effects)
 - Layered graph structure with fact and action levels
 - Fact level (F level): positive atoms (the first level being the initial state)
 - Action level (A level): actions that can be applied using the atoms in the previous fact level
 - Links: precondition and effect links between the two layers
- Record conflicts caused by negative effects and propagate them
- Extract a plan by choosing only non-conflicting parts of the graph (allowing for parallel actions)
- Parallelism (for non-conflicting actions) is a great boost for the efficiency.

Example Graph

- $\frac{\mathbf{I} = \{at(p1, c), at(p2, s), at(t1, c)\}}{\mathbf{G} = \{at(p1, s), in(p2, t1)\}},$
- All applicable actions are included
- In order to propagate unchanged properties, use noop action, denoted by *
- Expand graph as long as not all goal atoms are in the fact level



- Start at last fact level with goal atoms
- Select a minimal set of non-conflicting actions that generate the goal atoms
 - Two actions are conflicting if they have complementary effects or if one action deletes or asserts a precondition of the other action
- Use the preconditions of the selected actions as (sub-)goals on the next lower fact level
- Backtrack if no non-conflicting choice is possible
- If all possibilities are exhausted, the graph has to be extended by another level.

Extracting From the Example Graph

Final selection



Idea: Try to identify as many pairs of conflicting choices as possible in order to prune the search space

- Any pair of conflicting actions is mutex (mutually exclusive)
- A pair of atoms is mutex at F-level *i* > 0 if all ways of making them true involve actions that are mutex at the A-level *i*
- A pair of actions is also mutex if their preconditions are
- ...
- $\rightarrow\,$ Actions that are mutex cannot be executed at the same time
- $\rightarrow\,$ Facts that are mutex cannot be both made true at the same time
 - Never choose mutex pairs during plan extraction

Plan graph search and mutex propagation make planning 1–2 orders of magnitude more efficient than conventional methods

Satisfiability-Based Planning based on Planning Graphs

- Use planning graph in order to generate Boolean formula
 - The initial facts in layer F0 and the goal atoms in layer Fk are true
 - Each fact in layer *Fi* implies the disjunction of the actions having the fact as an effect
 - Each action implies the conjunction of the preconditions of the action
 - Conflicting actions cannot be executed at the same time.
- Turns out to be empirically more efficient than the earlier coding (because plans can be much shorter)
- Other codings are possible, e.g., purely action- or state-based codings

Disadvantages of Iterative Deepening Planners

- If a domain contains many symmetries, proving that there is no plan up to length of *k* − 1 can be very costly.
- Example: Gripper domain:
 - there is one robot with two grippers
 - there is room A that contains n balls
 - there is another room *B* connected to room *A*
 - the goal is to bring all balls to room B
- Obviously, the plan must have a length of at least n/2, but ID planners will try out all permutations of actions for shorter plans before noting this.
- Give better guidance

Heuristic Search Planning

- Use an heuristic estimator in order to select the next action or state
- Depending on the search scheme and the heuristic, the plan might not be the shortest one
- \rightarrow It is often easier to go for sub-optimal solutions (remember *Logistics*)



Heuristic search planner vs. iterative deepening on Gripper

- One can use progression or regression search, or even search in the space of incomplete partially ordered plans
- One can use local or global, systematic search strategies
- One can use different heuristics, which can be compared along the dimension of being
 - efficiently computable, i.e., should be computable in poly. time
 - informative, i.e., should make reasonable distinctions between search nodes
 - and admissible, i.e., should underestimate the real costs (useful in *A** search).

- Consider all states that are reachable by executing one action
- Try to improve the heuristic value
- Hill climbing: Select the successor with the minimal heuristic value
- Enforced hill climbing: Do a breadth-first search until you find a node that has a better evaluation than the current one.
- $\rightarrow\,$ Note: Because these algorithms are not systematic, they cannot be used to prove the absences of a solution

- Maintain a list of open nodes and select always the one which is best according to the heuristic
- Weighted A*: combine estimate h(S) for state S and costs g(S) for reaching S using the weight w with 0 ≤ w ≤ 1:

$$f(S) = w * g(S) + (1 - w) * h(S).$$

- If w = 0.5, we have ordinary A*, i.e., the algorithm finds the shortest solution provided h is admissible, i.e., the heuristics never overestimates
- If w < 0.5, the algorithm is greedy
- If w > 0.5, the algorithm behaves more like best-first search

- General principle for deriving heuristics:
 - Define a simplification (relaxation) of the problem and take the difficulty of a solution for the simplified problem as an heuristic estimator
- Example: straight-line distance on a map to estimate the travel distance
- Example: decomposition of a problem, where the components are solved ignoring the interactions between the components, which may incur additional costs
- In planning, one possibility is to ignore negative effects

Ignoring Negative Effects: Example

- In Logistics: The negative effects in *load* and *drive* are ignored:
- Simplified load operation: load(O, V, P)Precondition: at(O, P), at(V, P), truck(V)Effects: $\neg at(O, P), in(O, V)$
- After loading, the package is still at the place and also inside the truck
- Simplified drive operation: drive(V, P1, P2)
 Precondition: at(V, P1), truck(V), street(P1, P2)
 Effects: ¬at(V, P1), at(V, P2)
- After driving, the truck is in two places!
- \rightarrow We want the length of the shortest relaxed plan $\rightsquigarrow h^+(s)$
 - How difficult is monotonic planning?

Assume that all effects are positive

- finding some plan is easy:
 - Iteratively, execute all actions that are executable and have not all their effects made true yet
 - If no action can be executed anymore, check whether the goal is satisfied
 - If not, there is no plan
 - Otherwise, we have a plan containing each action only once
- Finding the shortest plan: easy or difficult?
- PLANLEN for precondition-free operators with only positive effects is NP-complete
- Consider approximations to *h*⁺.

The HSP Heuristic

- The first idea of estimating the distance to the goal for monotonic planning might be to count the number of unsatisfied goals atoms
- Neither admissible nor very informative
- Estimate the costs of making an atom *p* true in state *S*:

$$h(S,p) = \left\{ \begin{array}{ll} 0 & \text{if } p \in S \\ \min_{a \in \mathbf{O}, p \in \textit{eff}^+(a)} (1 + \max_{q \in \textit{pre}(a)} h(S,q)) & \text{otherwise} \end{array} \right.$$

- Estimate distance from *S* to *S*': $h(S, S') = max_{p \in S'}h(S, p)$
- Is admissible, because only the longest chain is taken, but it is not very informative
- Use \sum instead of max (this is the HSP heuristics)
- Is not admissible, but more informative. However, it ignores positive interactions!
- $\rightarrow\,$ Can be computed by using a dynamic programming technique

- Use the planning graph method to construct a plan for the monotone planning problem
- Can be done in poly. time (and is empirically very fast)
- Generates an optimal parallel plan that might not be the best sequential plan
- \rightarrow The number of actions in this plan is used as the heuristic estimate (more informative than the parallel plan length, but not *admissible*)
 - Appears to be a good approximation

- FF (Fast Forward) is a heuristic search planner developed in Freiburg
- Heuristic: Goal distances are estimated by solving a relaxation of the task in every search state (ignoring negative effects) the solution is not minimal, however!
- Search strategy: Enforced hill-climbing
- Pruning: Only a fraction of each states successors are considered: only thosesuccessors that would be generated by the relaxed solution – with a fall-back strategy considering all successors if we are unsuccessful
- FF is one of the fastest planners around
- $\rightarrow\,$ Meanwhile, faster systems such as FDD and LAMA, also designed in our group

Runtime: Logistics in the 2000 competition



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Foundations of AI

Solution Quality: Logistics in the 2000 competition



- FF was the fastest planner at the competition in 2000 across all planning domains and still is a benchmark system
- Further experiments showed that this extends to most other planning domains in the literature
- What is the search space topology under the used heuristic estimator?
- Problematical issues in the search space topology:
 - local minima
 - benches
 - dead ends

We have to go "upwards" before we can leave



All neighboring states look the same



There is no path to a solution



Classification of Benchmark Domains



These properties have been analytically proven for h^+ , but apply empirically also to the FF heuristic

(University of Freiburg)

Foundations of AI

Summary

- Rational agents need to plan their course of action
- In order to describe planning tasks in a domain-independent, declarative way, one needs planning formalisms
- Basic STRIPS is a simple planning formalism, where actions are described by their preconditions in form of a conjunction of atoms and the effects are described by a list of literals that become true and false
- PDDL is the current "standard language" that has been developed in connection with the international planning competition
- Basic planning algorithms search through the space created by the transition system or through the plan space.
- Planning with STRIPS using first-order terms is undecidable
- Planning with propositional STRIPS is PSPACE-complete
- Since 1992, we have reasonably efficient planning method for propositional, classical STRIPS planning
 - planning as satisfiability
 - the planning graph method
 - heuristic search planning (best method for non-optimal planning)