Bayes’ Rule Example

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1 What does the test \( t \) for an illness \( i \) tell me?

Given we have:

- 1% of the population is ill: \( p(i) = 0.01 \)
- Given an ill person, the test is positive in 90% of the cases: \( p(t \mid i) = 0.9 \)
- Given a person that is not ill, the test is positive in 20% of the cases: \( p(t \mid \neg i) = 0.2 \)

What is the probability of being ill given a positive test?

\[
p(i \mid t) = \frac{p(t \mid i)p(i)}{p(t)} \quad (1)
\]

\[
= \frac{p(t \mid i)p(i)}{\sum_i p(t \mid i)p(i)} \quad (2)
\]

\[
= \frac{p(t \mid i)p(i)}{p(t \mid i)p(i) + p(t \mid \neg i)p(\neg i)} \quad (3)
\]

\[
= \frac{p(t \mid i)p(i)}{p(t \mid i)p(i) + p(t \mid \neg i)(1 - p(i))} \quad (4)
\]

\[
= \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.2 \times 0.99} \quad (5)
\]

\[
= \frac{0.009}{0.207} \quad (6)
\]

\[
\approx 0.043 \quad (7)
\]

\[
\approx 4\% \quad (8)
\]

The probability of being ill is only 4%!