Albert-Ludwigs-Universität Freiburg
Lecture: Introduction to Mobile Robotics
Summer term 2012

Institut für Informatik

Prof. Dr. Wolfram Burgard
PD Dr. Cyrill Stachniss
Juniorprof. Dr. Maren Bennewitz
Juniorprof. Dr. Kai Arras

Sheet 5

Topic: Motion Models, Probability Distributions
Submission deadline: June 6, 2012
Submit to: `mobilerobotics@informatik.uni-freiburg.de`

Exercise 1: Expectation Value and Variance

Remark: This exercise is to be solved without Octave.

For the following probability distributions calculate the expectation value and the variance.
Hint: For a random variable X that is distributed according to the probability density $p(x)$, the expectation value of X is $E[X] = \int_{-\infty}^{+\infty} xp(x)dx$, and the expectation value of X^2 is $E[X^2] = \int_{-\infty}^{+\infty} x^2 p(x)dx$. The variance is given by $\sigma^2 = E[X^2] - E[X]^2$.

(a) Uniform distribution over the interval $[-c, c]$, $c \in \mathbb{R}_{\geq 0}$:

$$p(x) = \begin{cases} \frac{1}{2c} & x \in [-c, c] \\ 0 & \text{else} \end{cases}$$

(b) Triangular distribution over $[-c, c]$, $c \in \mathbb{R}_{\geq 0}$:

$$p(x) = \begin{cases} \frac{c+x}{c^2} & x \in [-c, 0] \\ \frac{c-x}{c^2} & x \in [0, c] \\ 0 & \text{else} \end{cases}$$

Exercise 2: Velocity-Based Motion Model

Remark: This exercise is to be solved without Octave.

Consider a robot which moves on a circular trajectory with noise-free constant velocities (v, w) (this situation is shown on page 30 of the *Probabilistic Motion Models* slides). The current pose of the robot is (x, y, θ).
(a) Derive the following expression for the center of the circle, (x_c, y_c):

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\frac{v}{w} sin\theta \\ \frac{v}{w} cos\theta \end{pmatrix}$$

(b) Now consider the situation where we are given a start pose (x, y, θ) and an end pose (x', y', θ'), connected by a circular movement. Prove that the center of the circle can be expressed as

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \frac{1}{2}\begin{pmatrix} x + x' \\ y + y' \end{pmatrix} + \mu \begin{pmatrix} y - y' \\ x' - x \end{pmatrix} \tag{1}$$

with some $\mu \in \mathbb{R}$

Hint: The circle lies on a ray that lies on the half-way point between (x, y) and (x', y') and is orthogonal to the line between these coordinates. Use the parametric equation for a line to represent this ray.

(c) Show that the value of μ is given by

$$\mu = \frac{1}{2}\frac{(x - x')cos\theta + (y - y')sin\theta}{(y - y')cos\theta - (x - x')sin\theta}.$$

Hint: μ can be calculated by using the fact that the line described by equation (1) and the line from (x_c, y_c) to (x, y) intersect at (x_c, y_c).

Exercise 3: Motion Model Implementation

A robot starting at pose $x_{t-1} = (x, y, \theta) = (0, 0, \frac{\pi}{2})$ moves with control command $u_t = (v, w) = (0.1\frac{m}{s}, 0.1\frac{rad}{s})$ during $t = 3$ seconds.

(a) Assuming an error-free movement, calculate the final pose $x_t = (x', y', \theta')$.

(b) Calculate the probability $p(x_t|u_t, x_{t-1})$ if we assume a noisy movement. Use the algorithm `motion_model_velocity` with parameters $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 2.5$.