Introduction to Mobile Robotics
Information Driven Exploration

Wolfram Burgard, Cyrill Stachniss,
Maren Bennewitz, Kai Arras
Tasks of Mobile Robots

- mapping
- localization
- exploration
- path planning

integrates approaches

active localization

SLAM
Exploration and SLAM

- SLAM is typically **passive**, because it consumes incoming sensor data
- Exploration **actively guides the robot** to cover the environment with its sensors
- Exploration in combination with SLAM: **Acting under pose and map uncertainty**
- Uncertainty should/needs to be taken into account when selecting an action
Mapping with Rao-Blackwellized Particle Filter (Brief Summary)

- Each particle represents a possible trajectory of the robot
- Each particle
  - maintains its own map and
  - updates it upon “mapping with known poses”
- Each particle survives with a probability proportional to the likelihood of the observations relative to its own map
Factorization Underlying Rao-Blackwellized Mapping

\[ p(x, m \mid z, u) \]

\[ = p(m \mid x, z, u)p(x \mid z, u) \]

Mapping with known poses

Particle filter representing trajectory hypotheses
Example: Particle Filter for Mapping

map of particle 1

3 particles

map of particle 2

map of particle 3
Outdoor Campus Map

- 30 particles
- 250x250m$^2$
- 1.75 km (odometry)
- 20cm resolution during scan matching
- 30cm resolution in final map
Combining Exploration and SLAM

- mapping
- localization
- path planning
- exploration
- integrated approaches
- active localization

SLAM
Exploration

- SLAM approaches seen so far are purely passive
- By reasoning about control, the mapping process can be made much more effective
- Question: *Where to move next?*
Where to Move Next?
Decision-Theoretic Approach

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

Utility = uncertainty reduction - cost
Example

high pose uncertainty
The Uncertainty of a Posterior

- **Entropy** is a general measure for the uncertainty of a posterior

\[
H(X) = - \int_x p(X = x) \log p(X = x) \, dx = E_X[- \log(p(X))]
\]

- **Conditional Entropy**

\[
H(X \mid Y) = \int_y p(Y = y)H(X \mid Y = y) \, dy
\]
Mutual Information

- **Expected Information Gain** or **Mutual Information** = Expected Uncertainty Reduction

\[
I(X;Y) = H(X) - H(X \mid Y)
\]

\[
I(X;Y) = H(Y) - H(Y \mid X)
\]

\[
I(X;Y \mid z = c_k) = H(X \mid z = c_k) - H(X \mid Y, z = c_k)
\]

\[
I(X;Y \mid Z) = H(X \mid Z) - H(X \mid Y, Z)
\]
Entropy Computation

\[ H(X,Y) \]
\[ = E_{X,Y}[- \log p(X,Y)] \]
\[ = E_{X,Y}[- \log (p(X)p(Y|X))] \]
\[ = E_{X,Y}[- \log p(X)] + E_{X,Y}[- \log p(Y|X)] \]
\[ = H(X) + \int_{x,y} -p(x,y) \log p(y|x) \, dx \, dx \]
\[ = H(X) + \int_{x,y} -p(y|x)p(x) \log p(y|x) \, dx \, dy \]
\[ = H(X) + \int_{x} px \int_{y} -p(y|x) \log p(y|y) \, dy \, dx \]
\[ = H(X) + \int_{x} p(x)H(Y|X=x) \, dx \]
The Uncertainty of the Robot

- The uncertainty of the RBPF:

\[ H(X, M) = H(X) + \sum_{i=1}^{\text{#particles}} \omega[i] H(M[i] | X[i] = x[i]) \]

- Trajectory uncertainty
- Particle weights
- Map uncertainty
Computing the Entropy of the Map Posterior

Occupancy Grid map $m$:

$$H(M) = - \sum_{c \in M} p(c) \log p(c) + (1 - p(c)) \log(1 - p(c))$$

- **map uncertainty**
- **grid cells**
- **probability that the cell is occupied**
Map Entropy

The overall entropy is the sum of the individual entropy values.
Computing the Entropy of the Trajectory Posterior

1. High-dimensional Gaussian

\[ H(G(\mu, \Sigma)) = \log((2\pi e)^{n/2} |\Sigma|) \]

reduced rank for sparse particle sets

2. Grid-based approximation

\[ H(X) \sim const. \]

for sparse particle clouds
Approximation of the Trajectory Posterior Entropy

Average pose entropy over time:

\[ H(X_{1:t} \mid d) \approx \frac{1}{t} \sum_{t'=1}^{t} H(X_{t'} \mid d) \]
Mutual Information

- The mutual information $I$ is given by the reduction of entropy in the belief

\[ I(X, M; Z^a) = \text{“uncertainty of the filter” - “uncertainty of the filter after carrying out action } a \text{”} \]
Integrating Over Observations

- Computing the mutual information requires to integrate over potential observations

\[ I(X, M; Z^a) = H(X, M) - H(X, M \mid Z^a) \]

\[ H(X, M \mid Z^a) = \int_{z} p(z \mid a) H(X, M \mid Z^a = z) \, dz \]
Integral Approximation

- The particle filter represents a posterior about possible maps

map of particle 1  map of particle 2  map of particle 3
Integral Approximation

- The particle filter represents a posterior about possible maps
- Simulate laser measurements in the maps of the particles

\[ H(X, M \mid Z^a) = \sum_{z} p(z \mid a)H(X, M \mid Z^a = z) \]

\[ = \sum_{i} \omega_i^H(X, M \mid Z^a = z_{sim}^i) \]
Simulating Observations

- Ray-casting in the map of each particle to generate observation sequences

map of particle $i$
The Utility

- We take into account the cost of an action: mutual information $\rightarrow$ utility $U$

- Select the action with the highest utility

$$a^* = \arg\max_a I(X, M; Z^a) - \text{cost}(a)$$
Focusing on Specific Actions

To efficiently sample actions we consider

- exploratory actions (1-3)
- loop closing actions (4) and
- place revisiting actions (5)
Dual Representation for Loop Detection

- **Trajectory graph** ("topological map") stores the path traversed by the robot
- **Occupancy grid** map represents the space covered by the sensors
- **Loops** correspond to long paths in the trajectory graph and short paths in the grid map
Example: Trajectory Graph
Application Example

high pose uncertainty
Example: Possible Targets
Example: Evaluate Targets

timestep 35

robot

![Diagram showing targets and expected utility](image)
Example: Move Robot to Target
Example: Evaluate Targets
Example: Move Robot

- Expected utility bars for target locations:
  - 0, 1, 2, 3, 4, 5, 6
- Decision at timestep 70
- Graph showing robot path from start to target location

... continue ...
Example: Entropy Evolution
Comparison

Map uncertainty only:

After loop closing action:
Real Exploration Example

Selected target location
Corridor Exploration

- The decision-theoretic approach leads to **intuitive behaviors**: “re-localize before getting lost”

- Some animals show a similar behavior (dogs marooned in the tundra of north Russia)
Summary

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions
- Experimental results demonstrate the usefulness of the overall approach