Albert-Ludwigs-Universität Freiburg
Lecture: Introduction to Mobile Robotics
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Sheet 4

Topic: Sampling, Motion Models, Sensor Models

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Exercise 1: Sampling

Implement three functions in Octave which generate samples of a normal distribution $\mathcal{N}(\mu, \sigma^2)$. The input parameters of these functions should be the mean μ and the variance σ^2 of the normal distribution. As only source of randomness, use samples of a uniform distribution.

- In the first function, generate the normal distributed samples by summing up 12 uniform distributed samples, as explained in the lecture.
- In the second function, use rejection sampling.
- In the third function, use the Box-Muller transformation method. The Box-Muller method allows to generate samples from a standard normal distribution using two uniformly distributed samples u_1, $u_2 \in [0, 1]$ via the following equation:
$$x = \cos(2\pi u_1)\sqrt{-2\log u_2}.$$

Compare the execution times of the three functions using Octave's built-in functions `tic` and `toc`. Also, compare the execution times of your own functions to the built-in function *normrnd*.

Exercise 2: Motion Model

A working motion model is a requirement for all Bayes Filter implementations. In the following, you will implement the simple odometry-based motion model.

(a) Implement the odometry-based motion model in Octave. Your function should take the following three arguments

$$\mathbf{x}_t = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \quad \mathbf{u}_t = \begin{pmatrix} \delta_{r1} \\ \delta_{r2} \\ \delta_t \end{pmatrix} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix},$$

where $\mathbf{x}_t$ is the current pose of the robot, $\mathbf{u}_t$ is the odometry reading obtaind from the robot, and $\boldsymbol{\alpha}$ are the noise parameters of the motion model. The return value of the function should be the new pose $\mathbf{x}_{t+1}$ of the robot.

As we do not expect the odometry measurements to be perfect, you will have to take the measurement error into account when implementing your function. Use the sampling methods you implemented in Exercise 1 to draw normally distributed random numbers for the noise in the motion model.

(b) If you evaluate your motion model over and over again with the same starting position, odometry reading, and noise values what is the result you would expect?

(c) Evaluate your motion model 5000 times for the following values

$$\mathbf{x}_t = \begin{pmatrix} 2.0 \\ 4.0 \\ 0.0 \end{pmatrix} \quad \mathbf{u}_t = \begin{pmatrix} \frac{\pi}{2} \\ 0.0 \\ 1.0 \end{pmatrix} \quad \boldsymbol{\alpha} = \begin{pmatrix} 0.1 \\ 0.1 \\ 0.01 \\ 0.01 \end{pmatrix}.$$

Plot the resulting (x, y) positions for each of the 5000 evaluations in a single plot.

Exercise 3: Distance-Only Sensor

In this exercise, you try to locate your friend using her cell phone signals. Suppose that in the map of Freiburg, the campus of the University of Freiburg is located at $m_0 = (10, 8)^T$, and your friend's home is situated at $m_1 = (6, 3)^T$. You have access to the data received by two cell towers, which are located at the positions $x_0 = (12, 4)^T$ and $x_1 = (5, 7)^T$, respectively. The distance between your friend's cell phone and the towers can be computed from the intensities of your friend's cell phone signals. These distance measurements are disturbed by zero-mean Gaussian noise with variances $\sigma_0^2 = 1$ for tower 0 and $\sigma_1^2 = 1.5$ for tower 1. You receive the distance measurements $d_0 = 3.9$ and $d_1 = 4.5$ from the two towers.

(a) At which of the two places is your friend more likely to be? Explain your calculations.

(b) Implement a function in Octave which generates a 3D-plot of the likelihood function which you used in a). Furthermore, mark m_0, m_1, x_0 and x_1 in the plot. Is the likelihood function which you plotted a probability density function? Give a reason for your answer.

(c) Now, suppose you have prior knowledge about your friend's habits which suggests that your friend currently is at home with probability $P(\text{at home}) = 0.7$, at the university with $P(\text{at university}) = 0.3$, and at any other place with $P(\text{other}) = 0$. Use this prior knowledge to recalculate the likelihoods of a).