Exercise 1: Counting Model

A robot applies the so-called simple counting approach to build a grid map of a 1D environment consisting of the cells \( c_0, \ldots, c_3 \). While standing in cell \( c_0 \), the robot integrates four measurements \( z_{t_0}, \ldots, z_{t_3} \). After integrating these measurements, the resulting belief of the robot with regards to the occupancy of the four cells is \( b_0 = 0, b_1 = \frac{1}{4}, b_2 = \frac{2}{3}, b_3 = 1 \). Given that the first three measurements are \( z_{t_0} = 1, z_{t_1} = 2, z_{t_2} = 3 \), compute the value of the last measurement \( z_{t_3} \).

Exercise 2: Occupancy Mapping

A robot has to build an occupancy grid map (cells \( c_0, \ldots, c_n \)) of a simple one-dimensional environment using a sequence of measurements from a range sensor.

Assume a very simple sensor model: every grid cell with a distance (based on its coordinate) smaller than the measured distance is assumed to be occupied with \( p = 0.3 \). Every cell behind the measured distance is occupied with \( p = 0.6 \). Every cell located more than 20\( \text{cm} \) behind the measured distance should not be updated. Calculate the resulting occupancy grid map using the inverse sensor model (see mapping lecture PDF, slide 10).

Use Octave. Use one array \( m = 0.5 \times \text{ones}(1, 21) \) for the belief values, and one array \( c = [0:10:200] \) for the cell coordinates. Use \( \text{plot}(c, m) \) to visualize the belief.
Exercise 3: Occupancy Mapping

Proove that in the occupancy grid mapping framework the occupancy value of a grid cell $P(m_j | x_{1:t}; z_{1:t})$ is independent of the order in which the measurements are integrated.