Introduction to Mobile Robotics

Graph-Based SLAM

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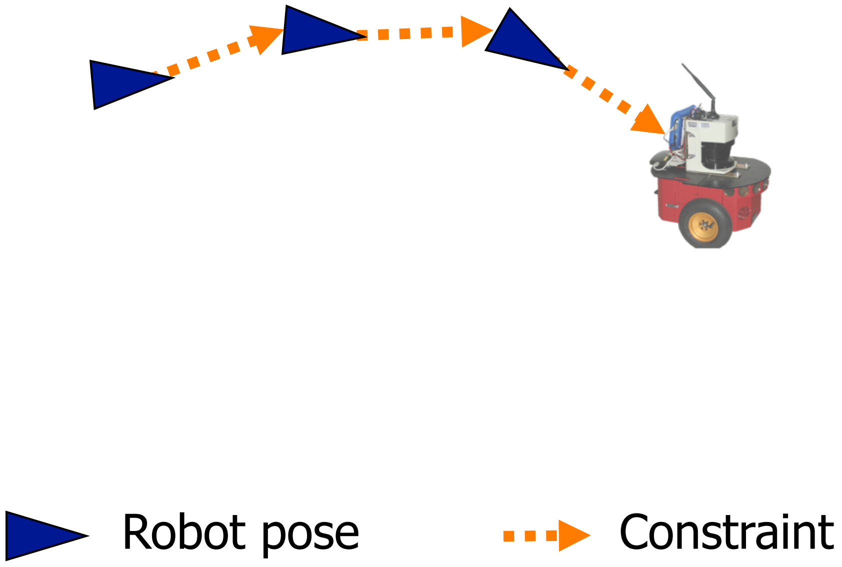
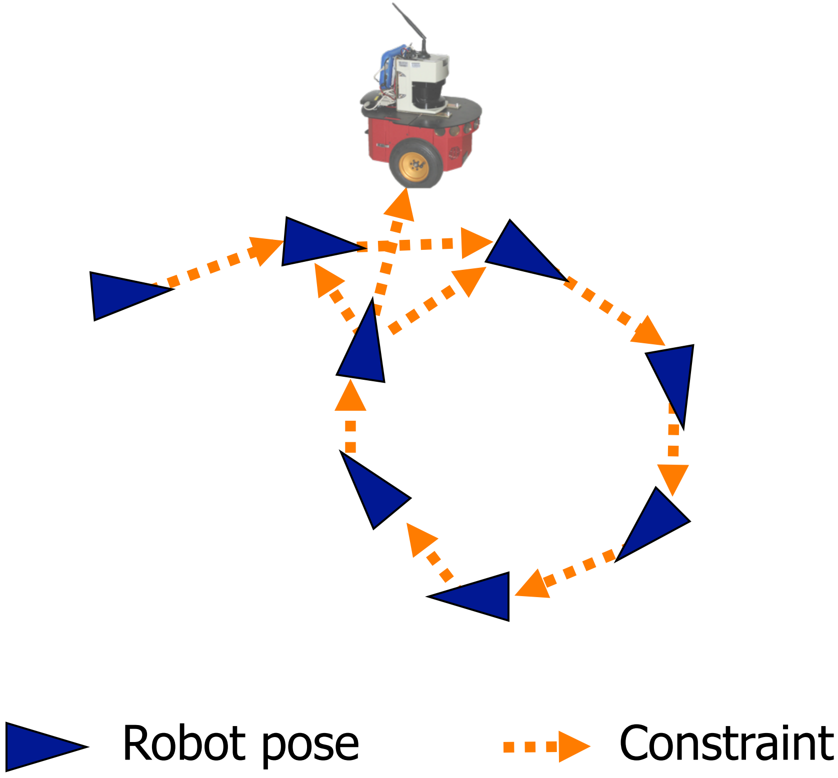
Graph-Based SLAM

- Constraints connect the poses of the robot while it is moving
- Constraints are inherently uncertain



Graph-Based SLAM

- Observing previously seen areas generates constraints between non-successive poses

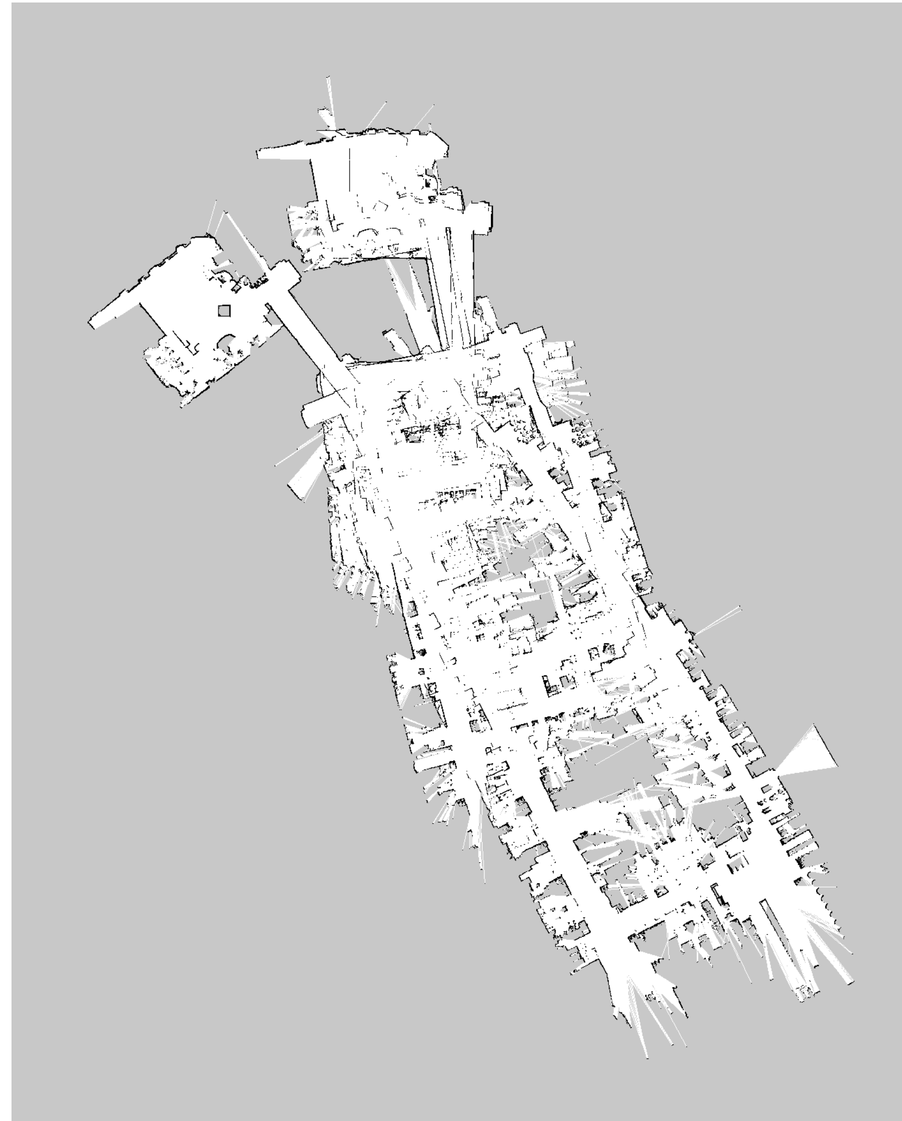


Idea of Graph-Based SLAM

- Use a **graph** to represent the problem
- Every **node** in the graph corresponds to a pose of the robot during mapping
- Every **edge** between two nodes corresponds to a spatial constraint between them
- **Graph-Based SLAM:** Build the graph and find a node configuration that minimize the error introduced by the constraints

Graph-Based SLAM in a Nutshell

- Every node in the graph corresponds to a robot position and a laser measurement
- An edge between two nodes represents a spatial constraint between the nodes



KUKA Halle 22, courtesy of P. Pfaff

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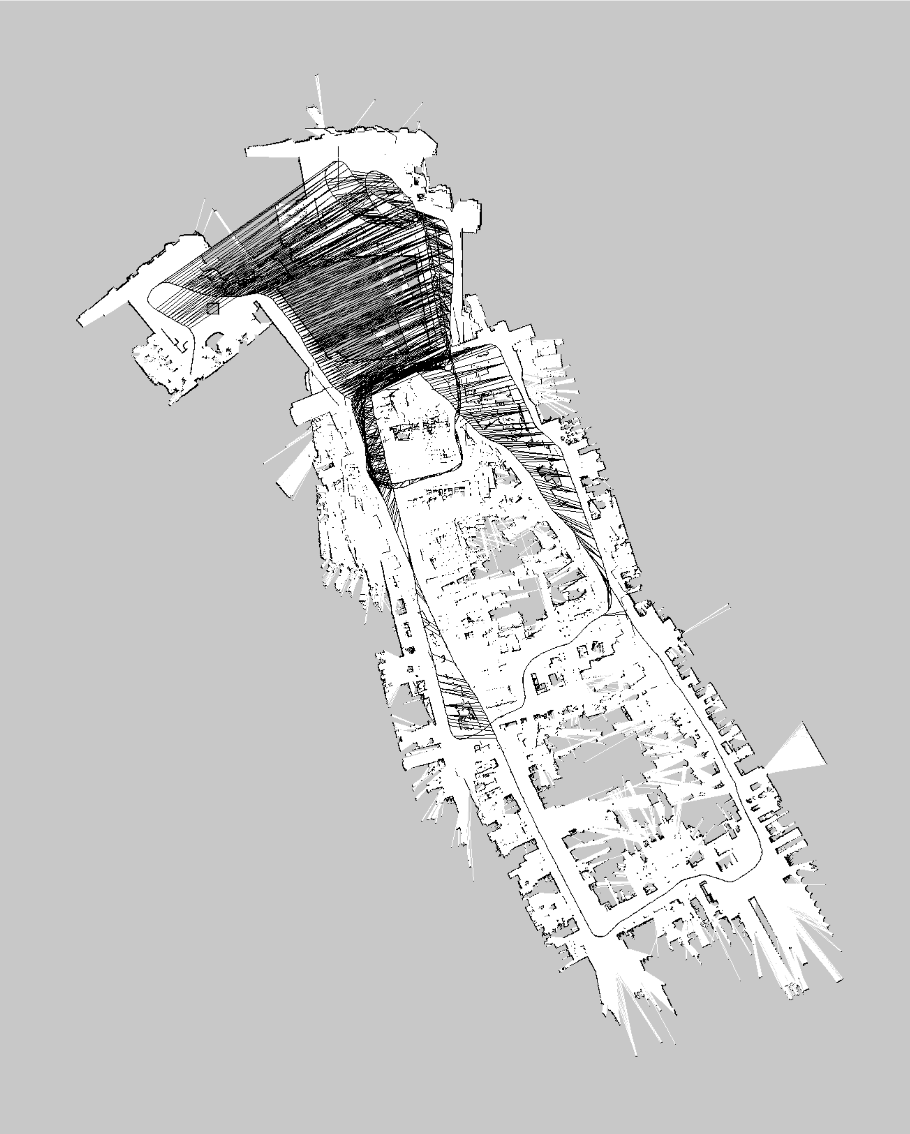

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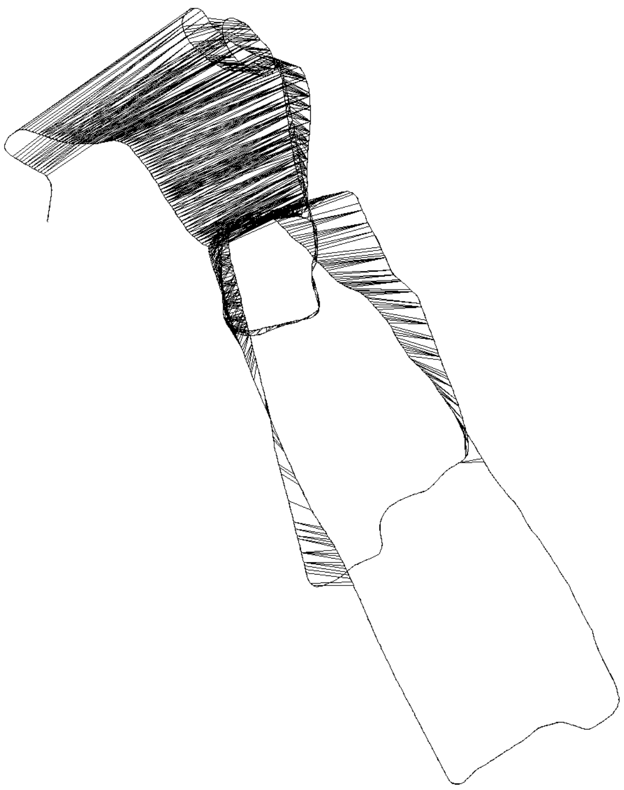
Graph-Based SLAM in a Nutshell

- Once we have the graph, we determine the most likely map by correcting the nodes



Graph-Based SLAM in a Nutshell

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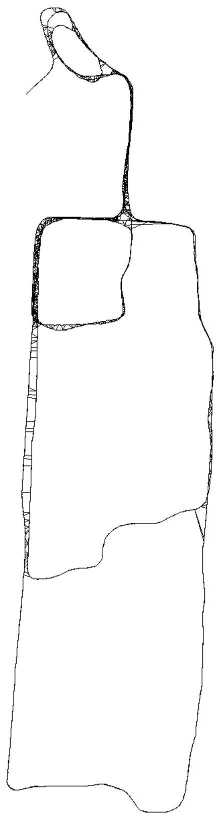
  … like this



Graph-Based SLAM in a Nutshell

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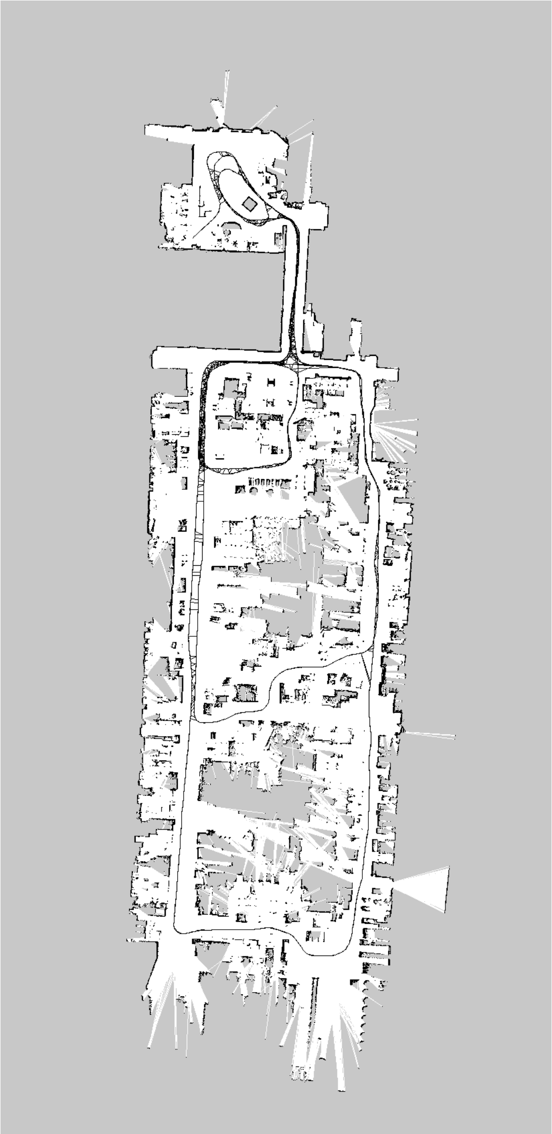
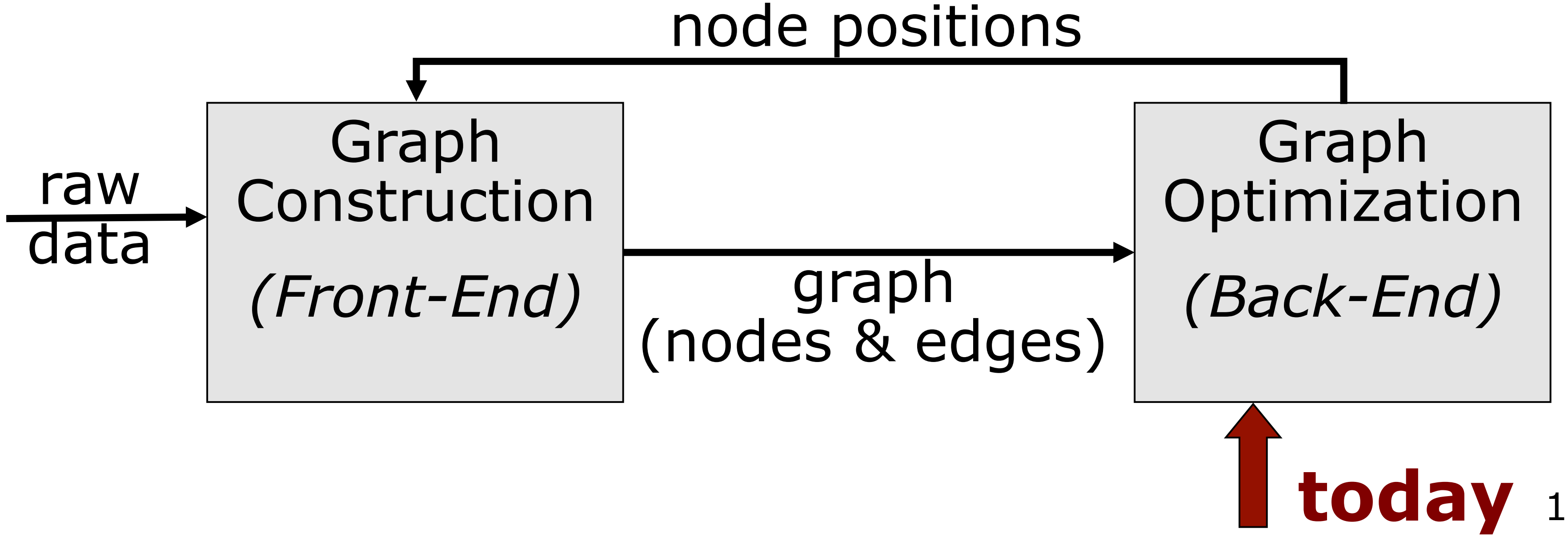
  … like this

- Then, we can render a map based on the known poses



The Overall SLAM System

- Interplay of front-end and back-end
- A consistent map helps to determine new constraints by reducing the search space
- This lecture focuses only on the optimization



Least Squares in General

- Approach for computing a solution for an **overdetermined system**
- “More equations than unknowns”
- Minimizes the **sum of the squared errors** in the equations
- Standard approach to a large set of problems

Problem

- Given a system described by a set of n observation functions $\{f_i(\mathbf{x})\}_{i=1:n}$
- Let
  - $\mathbf{x}$ be the state vector
  - $\mathbf{z}_i$ be a measurement of the state $\mathbf{x}$
  - $\widehat{\mathbf{z}}_i = f_i(\mathbf{x})$ be a function which maps $\mathbf{x}$ to a predicted measurement $\widehat{\mathbf{z}}_i$
- Given n noisy measurements $\mathbf{z}_{1:n}$ about the state $\mathbf{x}$

Goal: Estimate the state $\mathbf{x}$ which bests explains the measurements $\mathbf{z}_{1:n}$

Graphical Explanation

$$\mathbf{x}$$

$$f_1(\mathbf{x}) = \hat{\mathbf{z}}_1 \qquad \mathbf{z}_1$$

$$f_2(\mathbf{x}) = \hat{\mathbf{z}}_2 \qquad \mathbf{z}_2$$

$$\cdots$$

$$f_n(\mathbf{x}) = \hat{\mathbf{z}}_n \qquad \mathbf{z}_n$$

state (unknown)

predicted measurements

real measurements

Error Function

- Error $\mathbf{e}_i$ is typically the **difference** between the **predicted and actual** measurement

$$\mathbf{e}_i(\mathbf{x}) \quad = \quad \mathbf{z}_i - f_i(\mathbf{x})$$

- We assume that the error has **zero mean** and is **normally distributed**
- Gaussian error with information matrix $\mathbf{\Omega}_i$
- The squared error of a measurement depends only on the state and is a scalar

$$e_i(\mathbf{x}) \quad = \quad \mathbf{e}_i(\mathbf{x})^T \mathbf{\Omega}_i \mathbf{e}_i(\mathbf{x})$$

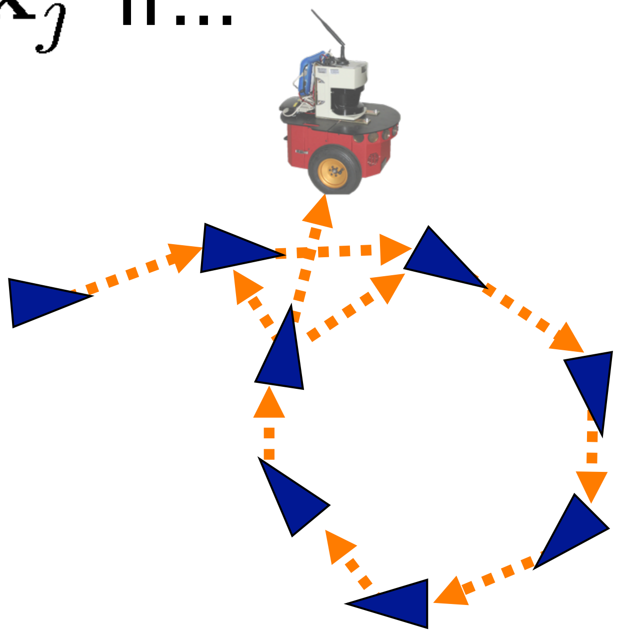
Least Squares for SLAM

- Overdetermined system for estimation the robot’s poses given observations
- “More observations than states”
- Minimizes the **sum of the squared errors**

Today: Application to SLAM

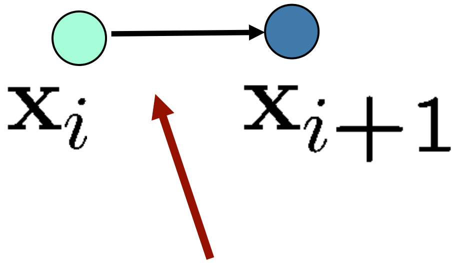
The Graph

- It consists of n nodes $\mathbf{x} = \mathbf{x}_{1:n}$
- Each $\mathbf{x}_i$ is a 2D or 3D transformation (the pose of the robot at time t_i)
- A constraint/edge exists between the nodes $\mathbf{x}_i$ and $\mathbf{x}_j$ if…



Create an Edge If… (1)

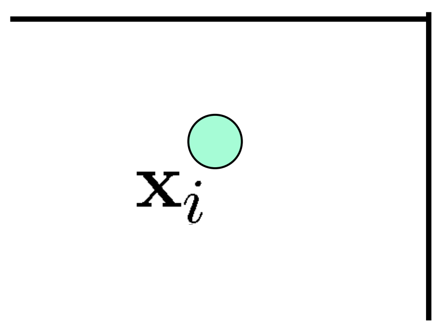
- …the robot moves from $\mathbf{x}_i$ to $\mathbf{x}_{i+1}$
- Edge corresponds to odometry

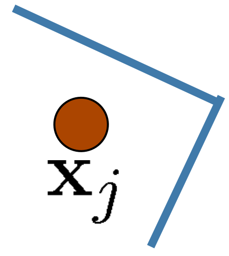


The edge represents the **odometry** measurement

Create an Edge If… (2)

- …the robot observes the same part of the environment from $\mathbf{x}_i$ and from $\mathbf{x}_j$
- Construct a **virtual measurement** about the position of $\mathbf{x}_j$ seen from $\mathbf{x}_i$



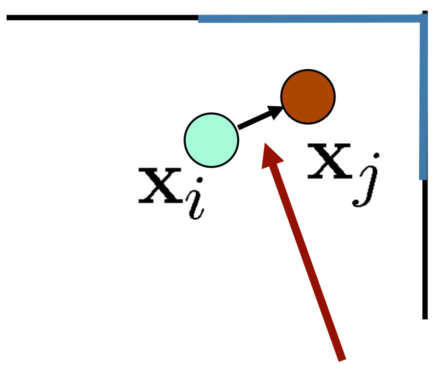


Measurement from $\mathbf{x}_i$

Measurement from $\mathbf{x}_j$

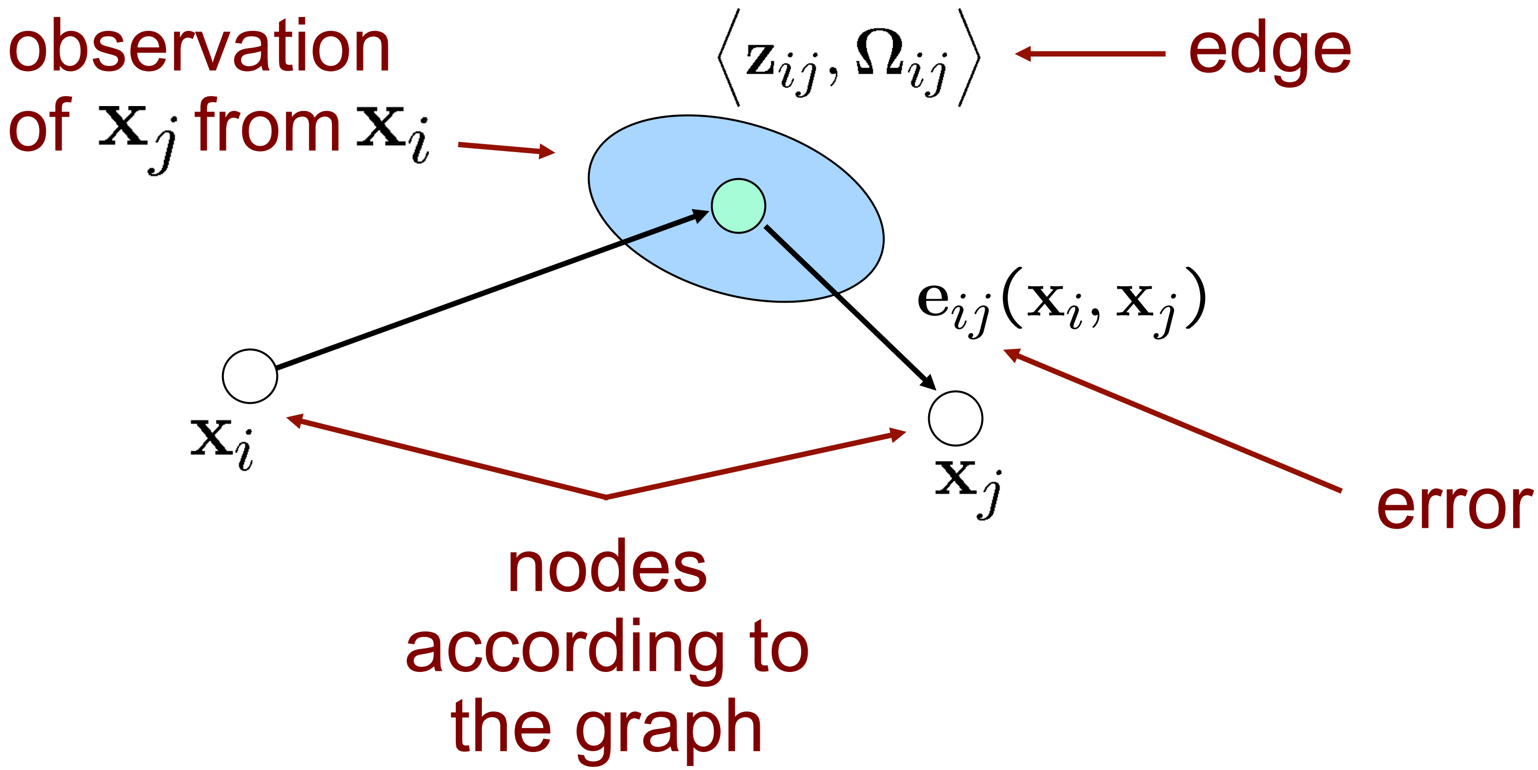
Create an Edge If… (2)

- …the robot observes the same part of the environment from $\mathbf{x}_i$ and from $\mathbf{x}_j$
- Construct a **virtual measurement** about the position of $\mathbf{x}_j$ seen from $\mathbf{x}_i$

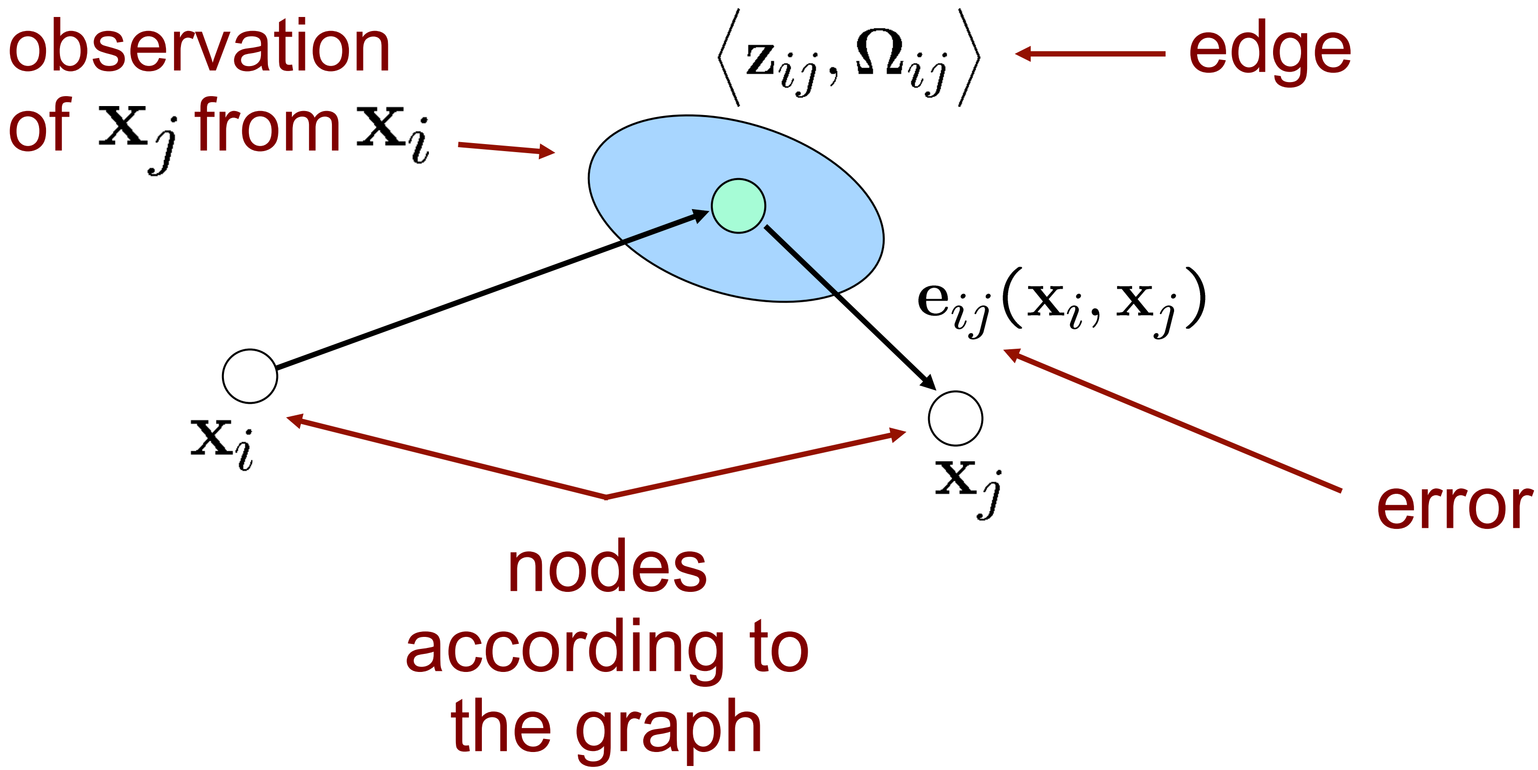


Edge represents the position of $\mathbf{x}_j$ seen from $\mathbf{x}_i$ based on the **observation**

Pose Graph


observation
of $\mathbf{x}_j$ from $\mathbf{x}_i$
$\langle \mathbf{z}_{ij}, \mathbf{\Omega}_{ij} \rangle$
edge
$\mathbf{e}_{ij}(\mathbf{x}_i, \mathbf{x}_j)$
$\mathbf{x}_i$
$\mathbf{x}_j$
error
nodes
according to
the graph

Pose Graph

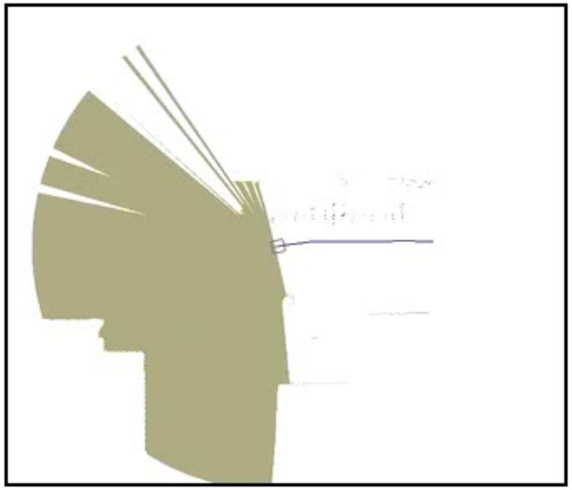


- **Goal:** $\mathbf{x}^* = \underset{\mathbf{x}}{\operatorname{argmin}} \sum_{ij} \mathbf{e}_{ij}^T \mathbf{\Omega}_{ij} \mathbf{e}_{ij}$

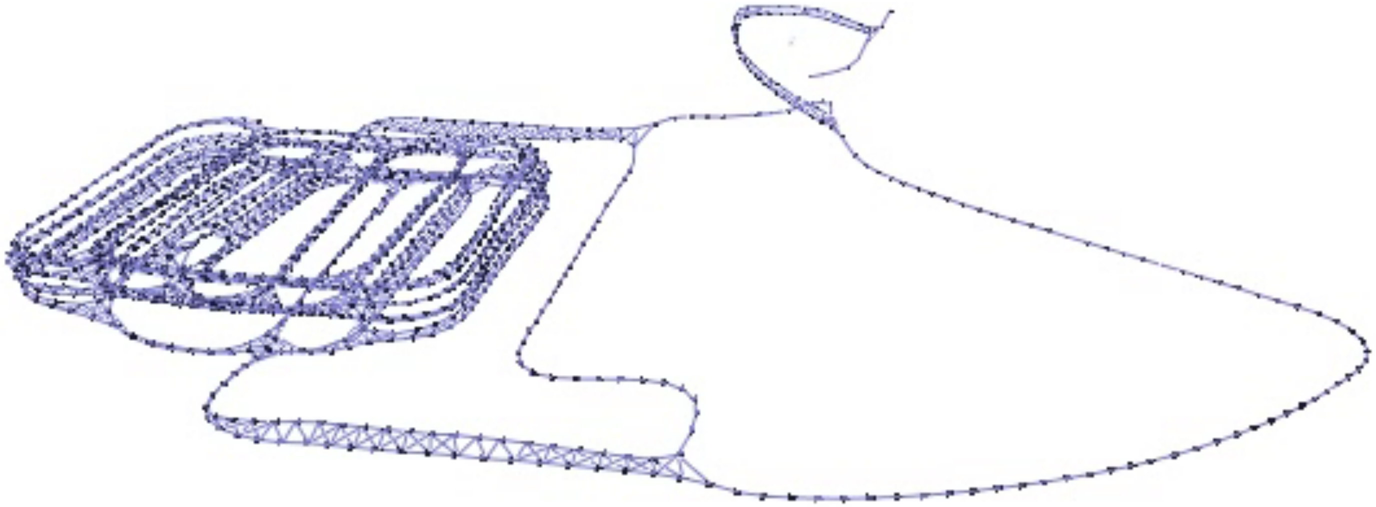
Gauss-Newton: The Overall Error Minimization Procedure

- Define the error function
- Linearize the error function
- Compute its derivative
- Set the derivative to zero
- Solve the linear system
- Iterate this procedure until convergence

Example: CS Campus Freiburg



Example: Stanford Garage



Conclusions

- The back-end part of the SLAM problem can be effectively solved with Gauss-Newton error minimization
- error functions computes the mis-match between the state and the observations
- One of the state-of-the-art solutions for computing maps