Introduction to Mobile Robotics

Information Driven Exploration

Wolfram Burgard, Maren Bennewitz, Diego Tipaldi, Luciano Spinello
Tasks of Mobile Robots

- mapping
- localization
- SLAM
- exploration
- integrated approaches
- path planning
- active localization
Exploration and SLAM

- SLAM is typically **passive**, because it consumes incoming sensor data
- Exploration **actively guides the robot** to cover the environment with its sensors
- Exploration in combination with SLAM: **Acting under pose and map uncertainty**
- Uncertainty should/needs to be taken into account when selecting an action
Mapping with Rao-Blackwellized Particle Filter (Brief Summary)

- Each particle represents a possible trajectory of the robot
- Each particle
  - maintains its own map and
  - updates it upon “mapping with known poses”
- Each particle survives with a probability proportional to the likelihood of the observations relative to its own map
Factorization Underlying Rao-Blackwellized Mapping

\[
p(x, m \mid z, u) = p(m \mid x, z, u)p(x \mid z, u)
\]

poses, map, observations & odometry

Mapping with known poses

Particle filter representing trajectory hypotheses
Example: Particle Filter for Mapping
Combining Exploration and SLAM
Exploration

- SLAM approaches seen so far are purely passive
- By reasoning about control, the mapping process can be made much more effective
- Question: **Where to move next?**
Where to Move Next?
Decision-Theoretic Approach

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

Utility = uncertainty reduction - cost
Example

high pose uncertainty
The Uncertainty of a Posterior

- **Entropy** is a general measure for the uncertainty of a posterior

\[
H(X) = -\int_X p(X = x) \log p(X = x) \, dx
= Ex[-\log(p(X))]
\]

- **Conditional Entropy**

\[
H(X \mid Y) = \int_Y p(Y = y)H(X \mid Y = y) \, dy
\]
Mutual Information

- **Expected Information Gain** or **Mutual Information** = Expected Uncertainty Reduction

\[
I(X;Y) = H(X) - H(X \mid Y)
\]

\[
I(X;Y) = H(Y) - H(Y \mid X)
\]

\[
I(X;Y \mid z = c_k) = H(X \mid z = c_k) - H(X \mid Y, z = c_k)
\]

\[
I(X;Y \mid Z) = H(X \mid Z) - H(X \mid Y, Z)
\]
Entropy Computation

\[ H(X,Y) \]
\[ = E_{X,Y}[-\log p(X,Y)] \]
\[ = E_{X,Y}[-\log(p(X) p(Y | X))] \]
\[ = E_{X,Y}[-\log p(X)] + E_{X,Y}[-\log p(Y | X)] \]
\[ = H(X) + \int_{x,y} -p(x,y) \log p(y | x) \, dx \, dx \]
\[ = H(X) + \int_{x,y} -p(y | x)p(x) \log p(y | x) \, dx \, dy \]
\[ = H(X) + \int_{x} p(x) \int_{y} -p(y | x) \log p(y | y) \, dy \, dx \]
\[ = H(X) + \int_{x} p(x) H(Y | X = x) \, dx \]
\[ = H(X) + H(Y | X) \]
The Uncertainty of the Robot

- The uncertainty of the RBPF:

\[ H(X, M) = H(X) + \int_x p(x)H(M \mid X = x) \, dx \]

\[ H(X, M) = H(X) + \sum_{i=1}^{\text{#particles}} \omega[i] H(M[i] \mid X[i] = x[i]) \]

- trajectory uncertainty
- particle weights
- map uncertainty
Computing the Entropy of the Map Posterior

Occupancy Grid map $m$:

$$H(M) = - \sum_{c \in M} p(c) \log p(c) + (1 - p(c)) \log(1 - p(c))$$

- map uncertainty
- grid cells
- probability that the cell is occupied
Map Entropy

The overall entropy is the sum of the individual entropy values.
Computing the Entropy of the Trajectory Posterior

1. High-dimensional Gaussian

\[ H(\mathcal{N}(\mu, \Sigma)) = \log((2\pi e)^{n/2}|\Sigma|) \]

reduced rank for sparse particle sets

2. Grid-based approximation

\[ H(X) \sim\text{const.} \]

for sparse particle clouds
Approximation of the Trajectory Posterior Entropy

Average pose entropy over time:

\[ H(X_{1:t} \mid d) \approx \frac{1}{t} \sum_{t'=1}^{t} H(X_{t'} \mid d) \]
Mutual Information

- The mutual information $I$ is given by the expected reduction of entropy in the belief action to be carried out.

$$I(X, M; Z^a) = \text{“uncertainty of the filter”} - \text{“uncertainty of the filter after carrying out action } a\text{”}$$
Integrating Over Observations

- Computing the mutual information requires to integrate over potential observations

\[
I(X, M; Z^a) = H(X, M) - H(X, M \mid Z^a)
\]

\[
H(X, M \mid Z^a) = \int_{Z} p(z \mid a) H(X, M \mid Z^a = z) \, dz
\]

potential observation sequences
Approximating the Integral

- The particle filter represents a posterior about possible maps

map of particle 1  map of particle 2  map of particle 3
Approximating the Integral

- The particle filter represents a posterior about possible maps
- Simulate laser measurements in the maps of the particles

\[
H(X, M \mid Z^a) = \sum_{z} p(z \mid a)H(X, M \mid Z^a = z)
\]

measurement sequences simulated in the maps
likelihood (particle weight)

\[
= \sum_i \omega^{[i]} H(X, M \mid Z^a = z^{[i]}_{sim a})
\]
Simulating Observations

- Ray-casting in the map of each particle to generate observation sequences
The Utility

- We take into account the cost of an action: mutual information $\rightarrow$ utility $U$

- Select the action with the highest utility

$$a^* = \arg\max_a I(X, M; Z^a) - \text{cost}(a)$$
Focusing on Specific Actions

To efficiently sample actions we consider

- **exploratory actions (1-3)**
- **loop closing actions (4)** and
- **place revisiting actions (5)**
Dual Representation for Loop Detection

- **Trajectory graph** ("topological map") stores the path traversed by the robot
- **Occupancy grid** map represents the space covered by the sensors
- **Loops** correspond to long paths in the trajectory graph and short paths in the grid map
Example: Trajectory Graph
Application Example

High pose uncertainty
Example: Possible Targets
Example: Evaluate Targets

timestep 35
Example: Move Robot to Target

![Diagram showing robot movement and decision chart at timestep 35.](image)
Example: Evaluate Targets
Example: Move Robot

- Expected utility at decision timestep 70

- Graph shows target location distribution

- Diagram illustrates robot movement from start to target location
Example: Entropy Evolution
Comparison

Map uncertainty only:

After loop closing action:
Real Exploration Example

Selected target location
Corridor Exploration

- The decision-theoretic approach leads to **intuitive behaviors**: “re-localize before getting lost”
- Some animals show a similar behavior (dogs marooned in the tundra of north Russia)
Summary

- A decision-theoretic approach to exploration in the context of RBPF-SLAM

- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain

- Reasons about measurements obtained along the path of the robot

- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions