Theoretical Computer Science (Bridging Course)

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Exercise Sheet 1

Due: 6th November 2014

Exercise 1.1 (Graphs)

Let $\mathcal{G} := (G, E)$ be an undirected graph. We define the *degree* of a vertex $g \in G$ to be the number of edges incident to g. We say that $\mathcal{G}$ is **k-regular** ($k \geq 0$) if each vertex $g \in G$ has degree k, or, equivalently, if every vertex is directly connected by an edge to exactly k other vertices.

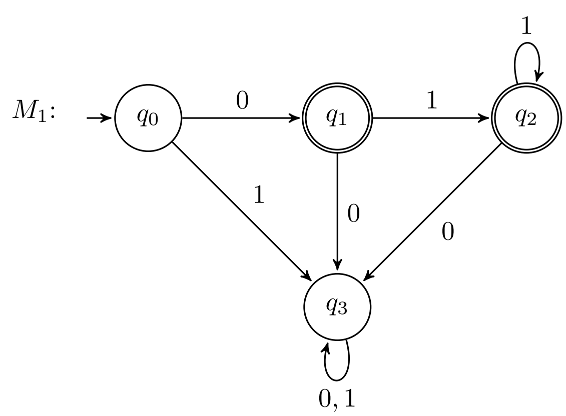
Prove that for every integer $k \geq 2$ there exists a k-regular graph $\mathcal{G}_k := (G_k, E_k)$ so that $|G_k| = 2k$ and diameter of $\mathcal{G}_k$ is 2.

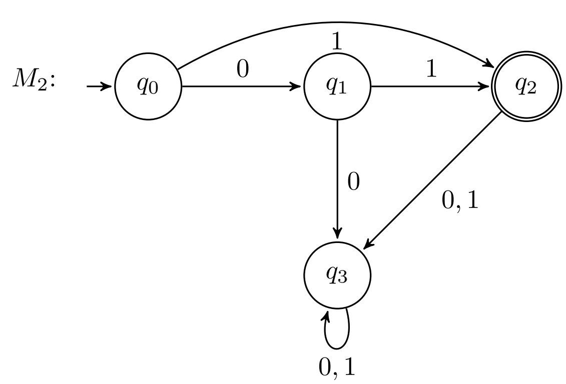
We recall that the *diameter* is the longest shortest path between any two vertices of the graph.

Hint: you can show the statement above either by construction or by induction.

Exercise 1.2 (DFA)

Consider the following two DFAs (deterministic finite automata) with $\Sigma = \{0, 1\}$:





(a) What languages (L_1 and L_2) do these two automata individually recognize?

(b) Give the formal definition for M_1.

(c) Show that $L_1 \cup L_2$ is also a regular language, by constructing **one** DFA. Please hand in a **high quality** diagram.

Exercise 1.3 (DFA)

(a) Construct a DFA that recognizes the language L with an alphabet $\Sigma = \{0, 1\}$, where $L = \{w \mid w \text{ has both an even number of 0's and an even number of 1's}\}$

(b) Give the state diagram for a DFA accepting the language $L = \{w \mid w \text{ starts with 1 and contains 10 or starts with 0 and contains the 01}\}$ The alphabet is $\Sigma = \{0, 1\}$.

Exercise 1.4 (Regular Language)

In this exercise we want to prove that regular languages are closed under intersection and under complement. The intersection of two languages is defined as $L_1 \cap L_2$. The complement of a language is defined as the set of all words in Σ^* which are not in L, i.e. $\bar{L} = \Sigma^* \setminus L$ (Σ^* is the set of all words/strings over Σ).
Let L and L' be regular languages that are recognized by DFAs $M = (Q, \Sigma, \delta, q_0, F)$ and $M' = (Q', \Sigma', \delta', q'_0, F')$, respectively.

(a) Show that the regular languages are closed under *intersection*, i.e. give a finite automaton that recognizes $L \cap L'$.

(b) Show that the regular languages are closed under *complement*, i.e. give a finite automaton that recognizes $\bar{L}$.