Exercise Sheet 6
Due: 11th December 2014

Exercise 6.1 (Turing Machines)
(a) Design a Turing Machine that decides the language \( L := \{0^n1^n \mid n \geq 1\} \). Explain your choice.
(b) Give the sequence of configurations for the input string 0011.
(c) Give the sequence of configurations for the input string 0010.

Exercise 6.2 (Turing Machines)
Describe a TM that decides the language
\[
L = \{w \in \{0, 1\}^* \mid |w|_0 = |w|_1\}
\]
where \(|w|_0\) and \(|w|_1\) are respectively the number of 0’s and 1’s in \(w\).

Exercise 6.3 (Pushdown Automata, Turing Machines)
How would one simulate a PDA on a Turing machine? Please do not write the Turing machine itself, but rather write the key idea in plain English.

Exercise 6.4 (Non Deterministic Turing Machines)
We call a natural number \textit{composite} if it is not prime\(^1\), formally, the set of natural composite numbers is
\[
\{hk \mid h, k \in \mathbb{N}, h, k \geq 2\}
\]
Give a nondeterministic Turing machine of the alphabet of vertical bars \(\Sigma = \{|\}\) that recognizes the language of composite numbers encoded as unary numbers (i.e. a natural number \(n\) is encoded in the form \(|n|\)). You should not give a formal construction, but describe the idea behind it as precise as possible.

\(^1\)Remember that 1 and \(-1\) are \textbf{not} prime numbers! By definition, prime numbers must not be invertible in \(\mathbb{Z}\).