

Theoretical Computer Science (Bridging Course)

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Exercise Sheet 7

Due: 18th December 2014

Exercise 7.1 (Decidable Languages)

Let L and L' be decidable languages. Prove the following properties.

- (a) The complement \bar{L} is decidable.
- (b) The union $L \cup L'$ is decidable.

Exercise 7.2 (Decidable Languages)

Show that the following languages are decidable:

- (a) $EQ_{DFA.RE} = \{\langle D, R \rangle \mid D \text{ is a DFA and } R \text{ is a regular expression and } L(D) = L(R)\}$
- (b) $A_{\epsilon CFG} = \{\langle G \rangle \mid G \text{ is a CFG that generates } \epsilon\}$
- (c) $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA that recognizes } \Sigma^*\}$

Exercise 7.3 (Undecidable Languages)

Consider the problem of determining whether a two-tape Turing machine ever writes a non-blank symbol on its second tape, i.e.

$$N = \{\langle M, w \rangle \mid M \text{ is a two-tape Turing machine which writes a non-blank symbol onto its second tape when it runs on } w\}.$$

Show that N is undecidable.

Hint: Use a reduction from A_{TM} .