Theoretical Computer Science (Bridging Course)

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Exercise Sheet 8

Due: 8th January 2014

Exercise 8.1 (Runtime)

You have implemented an algorithm that needs exactely $f(n)$ steps to terminate, where n is the size of the input. Assume that on your machine each step takes $1\mu s$.

For which maximal input size does your algorithm terminate within *one* day? Which input size can it maximally process in 10 days? Answer these (two!) questions for the following runtimes:

(a) $f(n) = n$

(b) $f(n) = n^2$

(c) $f(n) = 2^n$

(d) $f(n) = n^2 + n$

(e) *(Extra, not mandatory)* $f(n) = n \log n$
Hint: To compute the value of f^{-1}, you can implement the bisection method.

Exercise 8.2 (Big-O)

Consider the Turing machine below. The input alphabet is $\Sigma = \mathbb{N} = \{1, 2, 3, \dots\}$. The operator $|w|$ denotes the length of the string w, the relation $<$ is the smaller relation on the natural numbers.

M = "On input string w":
for $i = 1$ to $|w|$
    for $j = |w|$ downto $i + 1$
        if $w_j < w_{j-1}$
            swap w_j and w_{j-1}
        endif
    endfor
endfor

Assume that the runtime of a swap and of a comparison of two natural numbers is constant.

(a) What is the smallest exponent $k \in \mathbb{R}$ so that the runtime of the Turing machine M is in $O(|w|^k)$? Justify your answer.

(b) What does M compute (i.e. what is written on the tape when M halts)?

Exercise 8.3 (Big-O)

Characterise the relationship between $f(n)$ and $g(n)$ in the following examples using the O, Θ or Ω-notation.

1. $f(n) = n^{0.99998}$ $g(n) = \sqrt{n}$
2. $f(n) = 2^{\log^2(n)}$ $g(n) = \sum_{k=1}^{n^2} \frac{n}{2^k}$
3. $f(n) = n \cdot log_2 n$ $g(n) = \sqrt[3]{n}$
4. $f(n) = \sqrt{n}$ $g(n) = 1000n$
5. *(Extra, not mandatory)* $f(n) = \frac{n^{n+1}}{(n+1)^n}$, $g(n) = \sqrt[n]{n!}$

Hint: Stirling's approximation could be useful here.