Theoretical Computer Science (Bridging Course)

Dr. G. D. Tipaldi
F. Boniardi
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University of Freiburg
Department of Computer Science

Exercise Sheet 12

Due: 5th February 2015

Exercise 12.1 (Resolution)

Consider the knowledge base $KB = \{A, B \vee E \vee \neg D, K \wedge E \leftrightarrow A \wedge B, \neg C \rightarrow D, E \vee F \rightarrow \neg D\}$. Use resolution to prove that $KB \models A \wedge C$.

Hint: According to *Contradiction Theorem*, $KB \models A \wedge C$ iff $KB \cup \{\neg(A \wedge C)\}$ is unsatisfiable.

Exercise 12.2 (Predicate Logic,Terminology)

Classify the following expressions as *terms*, *ground terms*, *atoms* and *formulae*. If there is more than one possibility for an expression, please list them all. In the expressions, a and b are constant symbols, x and y are variable symbols, f and g are function symbols, and P and Q are relation symbols.

(a) $\mathrm{P}(x, y)$

(b) $\mathrm{f}(\mathrm{a}, \mathrm{b})$

(c) $\mathcal{I} \models \mathrm{P}(\mathrm{a}, \mathrm{f}(\mathrm{b}))$

(d) $\mathcal{I}, \alpha \models \mathrm{P}(\mathrm{a}, \mathrm{f}(x))$

(e) $\mathrm{f}(\mathrm{g}(x), \mathrm{b})$

(f) $\mathrm{Q}(x)$ is satisfiable.

(g) $\exists x(\mathrm{P}(x, y) \wedge \mathrm{Q}(x)) \vee \mathrm{P}(y, x)$

(h) $\forall x(\exists y(\mathrm{P}(x, y) \wedge \mathrm{Q}(x)) \vee \mathrm{P}(x, y))$

(i) $\forall x \forall y(\mathrm{P}(x, y) \wedge \mathrm{Q}(x) \vee \mathrm{P}(\mathrm{f}(y), x))$

(j) $\mathrm{Q}(x) \vee \mathrm{P}(x, y) \equiv \mathrm{P}(x, y) \vee \mathrm{Q}(x)$

Exercise 12.3 (**Extra**, Predicate Logic, Interpretation)

Consider the following set of formulae:

$$KB = \left\{ \begin{array}{l} \forall x \neg \mathrm{P}(x, x) \\ \forall x \forall y \forall z((\mathrm{P}(x, y) \wedge \mathrm{P}(y, z)) \rightarrow \mathrm{P}(x, z)) \\ \forall x \forall y(\mathrm{P}(x, y) \vee (x = y) \vee \mathrm{P}(y, x)) \end{array} \right\}$$

- Specify an interpretation $\mathcal{I} = \left\langle \mathcal{D}, \cdot^{\mathcal{I}} \right\rangle$ with $\mathcal{D} = \{d_1, \ldots, d_4\}$ and prove that $\mathcal{I} \models KB$ (i.e., $\mathcal{I} \models \varphi$ for all $\varphi \in KB$). Why is it not necessary to specify a variable assignment α to state a model of KB?