Revision Sheet

Question 1 (Finite Automata, 8 + 6 points)

(a) Give a regular expression for each of the following languages:

(i) all strings over \{0, 1\} that are at least three symbols long and have a 0 at their resp. 3rd positions

(ii) all strings over \{0, 1\} that have odd length, if starting with a 0, and even length otherwise

(iii) all strings over \{a, b\} that contain the substrings \textit{aa} or \textit{bab}

(iv) all strings over \{a, b\} that do not contain the substring \textit{ba}

(b) Draw a DFA equivalent to each of the following regular expressions:

(i) \(a(a \cup b)^*b\)

(ii) \((ab)^*\)

Question 2 (Regular languages, 14 points)
Let \(\Sigma = \{a, b\}\). Use the pumping lemma to prove that:

\[ L = \{a^n b^{2n} a^{3n} \mid n \geq 0\} \]

is not regular.

Any other proof techniques will not receive any points.

Question 3 (Context-free languages, 7+7 points)

(a) Give the state diagram of a PDA recognizing the language

\[ A = \{a^i b^j \mid i > 0 \text{ and } j = i + 1\} \]

(b) Let \(G = \langle \{S, X, Y, Z\}, \{a, b, c, d\}, R, S \rangle\) be the CFG with rules:

\[
S \rightarrow XYZ \\
X \rightarrow Xa \mid b \mid \varepsilon \\
Y \rightarrow b \mid c \\
Z \rightarrow cd
\]

Specify a CFG \(G_0\) in Chomsky Normal Form such that \(L(G_0) = L(G)\).
Question 4 (NP-completeness, 7 + 7 points)

Let $G := (V, E)$ be an undirected graph. A vertex cover of $G$ is a vertex set $C \subseteq V$ such that for all $(u, v) \in E$, $u \in C$ or $v \in C$.

Let $S := (S, C)$ be a subset collection, i.e., $S$ is a finite set and $C = \{C_1, \ldots, C_n\}$ where $C_i \subseteq S$ for all $i \in \{1, \ldots, n\}$. A hitting set of $S$ is a subset $H \subseteq S$ such that $H \cap C_i \neq \emptyset$ for all $i \in \{1, \ldots, n\}$.

The VertexCover and HittingSet decision problems are defined as:

- **VertexCover** = $\{\langle G, n \rangle | G$ is a graph which has a vertex cover of size at most $n \in \mathbb{N}_1\}$
- **HittingSet** = $\{\langle S, m \rangle | S$ is a subset collection with a hitting set of size at most $m \in \mathbb{N}_1\}$

(a) Prove that VertexCover $\leq_p$ HittingSet.

(b) Prove that HittingSet is NP-complete. (You may use your result from part (a) and that it is known that VertexCover is NP-complete.)

Question 5 (Decidability, 4 + 10 points)

Consider the problem of testing whether a given single-tape Turing machine ever writes a blank symbol over a non-blank symbol during the course of its computation, for any input string.

(a) Formulate this problem as a language.

(b) Show that the problem is undecidable.

Question 6 (Propositional Logic, 5 + 9 points)

(a) Resolution is not a complete proof method. However, the contradiction theorem can be used to obtain a sound and complete method based on resolution for answering queries of the form “Does $KB \models \varphi$?”.

Describe how this is done in general, i.e., to which set of clauses the resolution method is applied, and which outcome of the resolution method means that $KB \models \varphi$.

You may assume that $KB$ is given as a set of clauses and $\varphi$ as a conjunction of literals.

(b) Use the method described in part (a) to prove $KB \models P \land R$ for

$KB = \{P \lor Q, \ P \lor Q \lor \neg R, \ P \lor R, \ Q \lor S, \ R, \ \neg R \lor S\}$.

Question 7 (Propositional logic, 9 + 5 points)

(a) Which of the following formulae are satisfiable? Which ones are valid? Which ones are unsatisfiable? For formulas belonging to several of these categories, please list all of them.

For all satisfiable cases, also provide a satisfying truth assignment. For the questions about validity and unsatisfiability, you do not need to justify your answers.

(i) $(A \lor \neg B) \rightarrow (A \land C)$

(ii) $(A \leftrightarrow B) \land (B \leftrightarrow \neg A)$

(iii) $(A \land B) \lor (\neg A \land \neg B)$
(iv) \((A \leftrightarrow B) \land (B \to \neg A)\)

(b) Prove that

\[(A \land B) \to C \equiv A \to (B \to C)\]

by providing a sequence of logical equivalences that transforms the left-hand side into the right-hand side.

**Question 8** (Example of multiple choice question)

In which of the following cases is the logical formula to the left a *reasonable formalization* of the natural-language sentence to the right?

- \(\forall x \forall y ((\text{LivesIn}(x, y) \land \neg \text{EatsUp}(x)) \to \text{BadWeatherIn}(y))\) “Whenever someone who lives in some place does not eat up, the weather in that place will be bad.”
- \(\forall x \forall y (\text{Friend}(x, y) \land \text{Friend}(y, x))\) “Whenever A is a friend of B, B is a friend of A.”
- \(\forall x \forall y (\text{FatherOf}(x, \text{me}) \land \text{DaughterOf}(y, x) \land \text{Female}(\text{me})) \to (y = \text{me})\) “If my father has a daughter and I am female, then that daughter is me.”
- \(\exists x \forall y \text{Father}(x, y)\) “Everybody has at least one father.”
- \(\text{DaughterOf}(\text{me}, \text{Friend})\) “I am the daughter of my friend.”