Theoretical Computer Science (Bridging Course)

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Exercise Sheet 2

Exercise 2.1 (Regular expressions)

Consider the following regular expressions. What language do they recognize? Give two strings that are members of the the corresponding language and two strings which are not members – a total of four strings for each part. Assume the alphabet $\Sigma = \{a, b, c\}$ in all parts.

(a) $a^*b^*c^*$

(b) $\Sigma^*aba\Sigma^*$

(c) $((a \cup c)^* b(a \cup c)^* b(a \cup c)^* b(a \cup c)^*)^*$

(d) $b \cup (b\Sigma^*b)$

Solution:

(a) $L = \{w \in \Sigma^* \mid$ the letters are in alphabetical order $\}$. For example, $aabb, aaaabbccc$ are members, whereas $acba, bbcab$ are not.

(b) $L = \{w \in \Sigma^* \mid$ contains $aba\}$. For example, $aabc, cababac$ are members, whereas abc, bb are not.

(c) $L = \{w \in \Sigma^* \mid$ the number of b is a multiple of $3\}$ For example, $abbbc, bbb$ are members, whereas abc, bb are not.

(d) $L = \{w \in \Sigma^* \mid w$ begins and ends in $b\}$ For example, $b, baaabab$ are members, whereas $ab, abba$ are not.

Exercise 2.2 (Regular Expressions)

Construct regular expressions for the following languages over the alphabet $\Sigma = \{0, 1\}$

(a) $L_1 = \{w \in \Sigma^* \mid$ every 0 in w is immediately followed by a $1\}$

(b) $L_2 = \{w \in \Sigma^* \mid$ the second or the third position from the end is a $1\}$

(c) $L_3 = \{w \in \Sigma^* \mid w$ consists of alternating 0 and $1\}$

(d) $L_4 = \{w \in \Sigma^* \mid w$ does not contain $11\}$

(e) $L_5 = \{w \in \Sigma^* \mid w$ ends in even number of $0s\}$

Solution:

(a) $re_1 = 1^* \cup (1^*011^*)^*$

(b) $re_2 = \Sigma^*1\Sigma \cup \Sigma^*1\Sigma\Sigma$

(c) $re_3 = (01)^* \cup (10)^* \cup 0(10)^* \cup 1(01)^*$

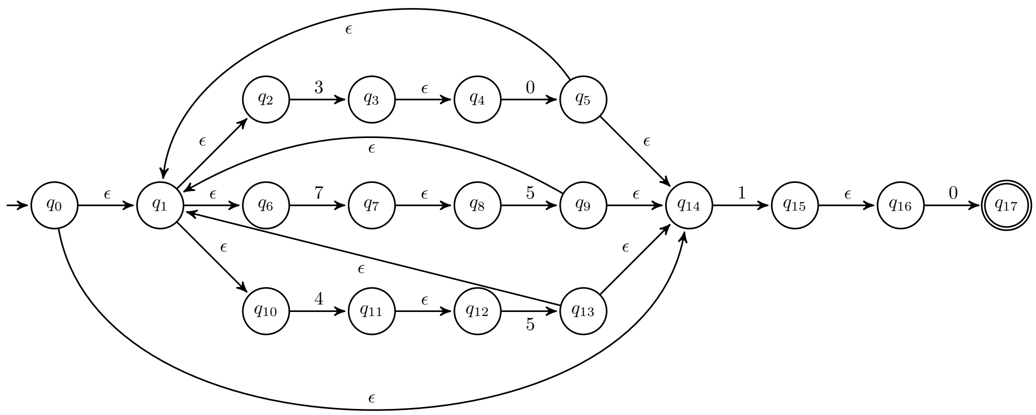
(d) $re_4 = 0^*((100^*)^*)(1 \cup 0^*)$

(e) $re_5 = (1 \cup 0)^*1(00)^* \cup (00)^*$

Exercise 2.3 (NFAs and Regular Expressions)

Consider the regular expression $(30 \cup 75 \cup 45)^* \circ 10$ (over the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$).

(a) Give the NFA that recognizes $L((30 \cup 75 \cup 45)^* \circ 10)$. *Solution*:



(b) Give another NFA with at most 5 states that recognizes the same language (you do not have to justify your answer).

Solution: