Theoretical Computer Science (Bridging Course)

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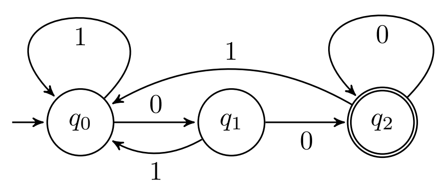
Exercise Sheet 4

Due: 27th November 2014

Exercise 4.1 (Context-free grammars, Chomsky normal form)

(a) Construct a context-free grammar for the following DFA:



Solution:
The language of the DFA is defined by the grammar $G = (V, \Sigma, R, S_0)$ with $V = \{S_0, S_1, S_2\}$, $\Sigma = \{0, 1\}$, and R being the following set of rules:

$$\begin{aligned} S_0 &\to 0S_1 \mid 1S_0 \\ S_1 &\to 0S_2 \mid 1S_0 \\ S_2 &\to 0S_2 \mid 1S_0 \mid \epsilon \end{aligned}$$

(b) Show that the grammar $(\{S\}, \{a, b\}, R, S)$ with rules $R = S \to aS \mid aSbS \mid \epsilon$ is ambiguous.
Solution:
Consider the string aab. We can give two different leftmost derivations of this string: $S \leadsto \mathbf{aS} \leadsto a\mathbf{aSbS} \leadsto aabS \leadsto aab$ and $S \leadsto \mathbf{aSbS} \leadsto a\mathbf{aS}bS \leadsto aabS \leadsto aab$.

(c) Give a grammar in Chomsky Normal Form that generates the same language as the grammar $G = (V, \Sigma, R, S)$ with $V = \{S, X, Y\}$, $\Sigma = \{a, b, c\}$, and R being the following set of rules:

$$\begin{aligned} S &\to XY \\ X &\to abb \mid aXb \mid \epsilon \\ Y &\to c \mid cY \end{aligned}$$

Solution:
Using the algorithm from the lecture, we get the grammar $G' = (V', \Sigma, R', S)$ with $V = \{S, X, X_1, X_2, Y, A, B, C\}$, $\Sigma = \{a, b, c\}$, and R' being the following set of rules:

$$\begin{aligned} S_0 &\to S \\ S &\to XY \\ X &\to abb \mid aXb \mid \epsilon \\ Y &\to c \mid cY \end{aligned}$$

$$\begin{aligned} S_0 &\to S \\ S &\to XY \mid Y \\ X &\to abb \mid aXb \mid ab \\ Y &\to c \mid cY \end{aligned}$$

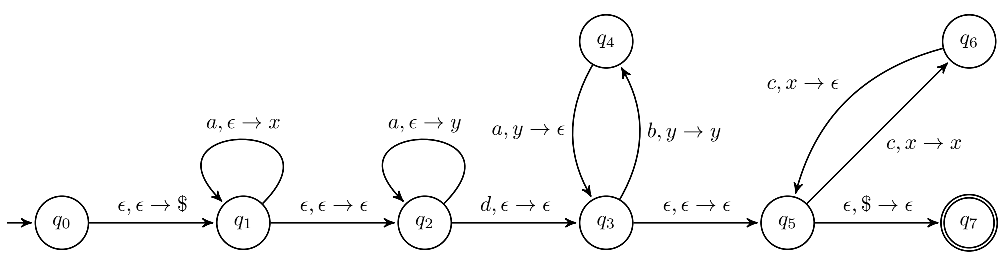
$$
\begin{aligned}
S_0 &\to S \\
S &\to XY \mid c \mid cY \\
X &\to abb \mid aXb \mid ab \\
Y &\to c \mid cY
\end{aligned}
$$

$$
\begin{aligned}
S_0 &\to S \\
S &\to XY \mid c \mid cY \\
X &\to aX_1 \mid aX_2 \mid ab \\
X_1 &\to bb \\
X_2 &\to Xb \\
Y &\to c \mid cY
\end{aligned}
$$

$$
\begin{aligned}
S &\to XY \mid c \mid CY \\
X &\to AX_1 \mid AX_2 \mid AB \\
X_1 &\to BB \\
X_2 &\to XB \\
Y &\to c \mid CY \\
A &\to a \\
B &\to b \\
C &\to c
\end{aligned}
$$

Exercise 4.2 (Pushdown Automata)

Consider the following PDA:



(a) Show that the PDA accepts the word *aaadbabacc*.

Solution:

$$
\begin{aligned}
&(aaadbabacc, q_0, \epsilon) \rightarrow \\
\rightarrow&(aaadbabacc, q_1, \$) \rightarrow \\
\rightarrow&(aadbabacc, q_1, x\$) \rightarrow \\
\rightarrow&(aadbabacc, q_2, x\$) \rightarrow \\
\rightarrow&(adbabacc, q_2, yx\$) \rightarrow \\
\rightarrow&(dbabacc, q_2, yyx\$) \rightarrow \\
\rightarrow&(babacc, q_3, yyx\$) \rightarrow \\
\rightarrow&(abacc, q_4, yyx\$) \rightarrow \\
\rightarrow&(bacc, q_3, yx\$) \rightarrow \\
\rightarrow&(acc, q_4, yx\$) \rightarrow \\
\rightarrow&(cc, q_3, x\$) \rightarrow \\
\rightarrow&(cc, q_5, x\$) \rightarrow \\
\rightarrow&(c, q_6, x\$) \rightarrow \\
\rightarrow&(\epsilon, q_5, \$) \rightarrow \\
\rightarrow&(\epsilon, q_7, \epsilon).
\end{aligned}
$$

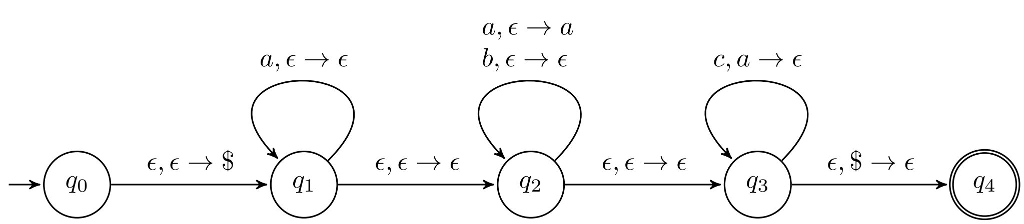
(b) Which language L does the given PDA accept?

Solution:
$L = \{a^n a^s d(ba)^s c^{2n} \in \{a, b, c, d\}^* \mid n, s \geq 0\}$

Exercise 4.3 (Pushdown Automata)
Create a PDA that recognizes the following context free language:

$$L = \{a^* w c^k \mid w \in \{a, b\}^* \text{ and } k = |w|_a \ (k = \text{the number of } a\text{s in } w)\}$$

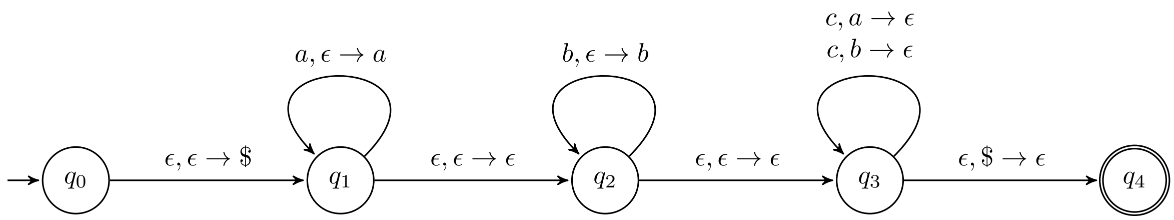
Solution:



Exercise 4.4 (Pushdown Automata)
Create a PDA that recognizes the following language.

$$L = \{a^i b^j c^k \mid i, j \geq 0, \ k = i + j\}$$

Solution:


$c, a \to \epsilon$
$c, b \to \epsilon$
$a, \epsilon \to a$
$b, \epsilon \to b$
$\epsilon, \epsilon \to \$$
$\epsilon, \epsilon \to \epsilon$
$\epsilon, \epsilon \to \epsilon$
$\epsilon, \$ \to \epsilon$
q_0
q_1
q_2
q_3
q_4