Foundations of Artificial Intelligence

Prof. Dr. W. Burgard, Prof. Dr. B. Nebel
J. Aldinger, J. Boedecker, C. Dornhege
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University of Freiburg
Department of Computer Science

Exercise Sheet 3

Due: Wednesday, June 17, 2015, before the lecture

Exercise 3.1 (Minimax algorithm)

(a) Perform the Minimax algorithm in the tree in Figure 1 using $\alpha\beta$-pruning. Traverse the tree from left to right. Annotate the nodes with their alpha and beta values.

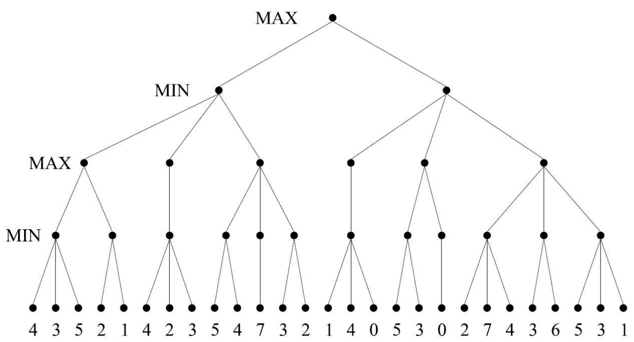
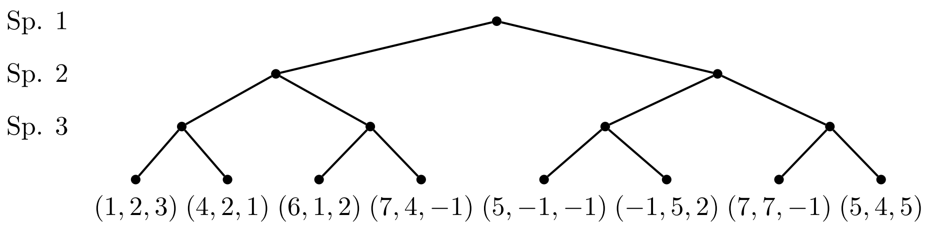


Figure 1: Minimax tree

(b) Can the nodes be ordered in such a way that $\alpha\beta$-pruning can cut off more branches? If so, give the order. Otherwise, argue why not.

(c) Consider the problem of search in a three-player game (you may assume that no alliances are allowed) without the zero-sum condition. The players are called 1, 2, and 3. Unlike in the case of two-player zero-sum games, the evaluation function now returns a triple (x_1, x_2, x_3) such that x_i is the value the node has for player i.

Complete the game tree given below by annotating all interior nodes and the root node with the backed-up value triples.



Exercise 3.2 (Satisfiability, Models)

(a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.

(i) $\mathit{Smoke} \Rightarrow \mathit{Smoke}$
(ii) $\mathit{Smoke} \Rightarrow \mathit{Fire}$
(iii) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow (\neg \mathit{Fire} \Rightarrow \neg \mathit{Smoke})$
(iv) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow ((\mathit{Smoke} \wedge \mathit{Heat}) \Rightarrow \mathit{Fire})$
(v) $\mathit{TheBestTeamWins} \Leftrightarrow \mathit{GermanyWinsWorldCup}$

(b) Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following formulae? Explain.

(i) $(A \wedge B) \vee (B \wedge C)$
(ii) $A \vee B$
(iii) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 3.3 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ, and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \tag{1}$$
$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \tag{2}$$
$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \tag{3}$$
$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \tag{4}$$
$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \tag{5}$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.

Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show that $K \models (\neg B \rightarrow (A \wedge C))$ holds.

Exercise 3.4 (Davis-Putnam Procedure)

Use the Davis-Putnam procedure to compute models for the following clause sets or to prove that no model exists. Whenever possible, apply the *pure symbol heuristic* (i.e. assignment of the corresponding value to variables always occurring with the same polarity) and *unit propagation*. At each step, indicate which rule you have applied.

(a) $\big\{\{P, \neg Q\}, \{\neg P, Q\}, \{Q, \neg R\}, \{S\}, \{\neg S, \neg Q, \neg R\}, \{S, R\}\big\}$
(b) $\big\{\{P, Q, S, T\}, \{P, S, \neg T\}, \{Q, \neg S, T\}, \{P, \neg S, \neg T\}, \{P, \neg Q\}, \{\neg R, \neg P\}, \{R\}\big\}$

Exercise 3.5 (Predicate Logic)
Consider following colloquial sentences:

(a) Not all students attend AI and ST.

(b) One student failed both AI and ST.

(c) Exactly two students failed ST.

(d) There is a barber who shaves all men in town who do not shave themselves.

(e) No one likes a professor who is not smart.

Represent these sentences in first-order logic using the predicates *student(x)*, *attends(x,y)*, *fails(x,y)*, *barber(x)*, *shaves(x,y)*, *professor(x)*, *likes(x,y)* und *smart(x)*.

Exercise 3.6 (Semantics of Predicate Logic)
Consider the Interpretation $\mathcal{I} =< \mathcal{D}, \cdot^{\mathcal{I}} >$ with

- $D = \{0, 1, 2, 3\}$
- $even^{\mathcal{I}} = \{0, 2\}$
- $odd^{\mathcal{I}} = \{1, 3\}$
- $lessThan^{\mathcal{I}} = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)\}$
- $two^{\mathcal{I}} = 2$
- $plus^{\mathcal{I}} : D \times D \to D, plus^{\mathcal{I}}(a, b) = (a + b) \mod 4$

and the variable assignment $\alpha = \{(x, 0), (y, 1)\}$.
Decide for the following formulae θ_i if $\mathcal{I}$ is a model for θ_i under α, i.e. if $\mathcal{I}, \alpha \models \theta_i$.
Explain your answer.

(a) $\theta_1 = odd(y) \wedge even(two)$

(b) $\theta_2 = \forall x\ (even(x) \vee odd(x))$

(c) $\theta_3 = \forall x \exists y\ lessThan(x, y)$

(d) $\theta_4 = \forall x\ (even(x) \Rightarrow \exists y\ lessThan(x, y))$

(e) $\theta_5 = \forall x\ (odd(x) \Rightarrow even(plus(x, y)))$

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.