Foundations of Artificial Intelligence

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Exercise Sheet 4
Due: Wednesday, July 1, 2015

Exercise 4.1 (Substitutions and Unification)

(a) Compute the substitutions
  (i) $P(x, y)\{\frac{x}{A}, \frac{y}{f(B)}\}$,
  (ii) $P(x, y)\{\frac{x}{f(y)}\}\{\frac{y}{g(B,B)}\}$,
  (iii) $P(x, y)\{\frac{x}{f(y)}, \frac{y}{g(B,B)}\}$ and
  (iv) $P(x, y)\{\frac{z}{f(B)}, \frac{x}{A}\}$

(b) Apply the unification algorithm to the following set of literals:
$\{R(h(x), f(h(u), y)), R(y, f(y, h(g(A))))\}$. In each step, give the values of T_k, s_k, D_k, v_k, and t_k.

Exercise 4.2 (Wumpus world and resolution)

Consider the following situation in the wumpus world:

The gray squares have already been visited, the others not. The percepts in the corresponding squares are indicated by *breeze* and *stench* .

(a) Formalize the general connections between breezes and pits using propositional formulae. Use 16 variables $P_{i,j}$ (meaning there is a *pit* in square $[i, j]$) and 16 variables $B_{i,j}$ (*breeze* in square $[i, j]$).

(b) Show, using *resolution*, that square $[3, 1]$ contains a pit in the given situation, i.e., show that KB $\models P_{3,1}$. The knowledge base KB consists of the propositions from part (a) as well as the percepts of the agent. Note: squares that already have been visited do not contain pits. If necessary, convert the knowledge base into CNF (conjunctive normal form).

Exercise 4.3 (Planning in the wumpus world)

Consider the following initial state in the wumpus world:

The agent in square $[1, 1]$ did not attend the "Action Planning" lecture, thus, he isn't able to solve planning tasks with partial observability. Additionally he is more excited about hunting the wumpus than about finding gold. Therefore, we define the planning problem as[1]:

Initial state $\mathcal{I}$:

$\{\texttt{connected}([1, 1], [2, 1]), \texttt{connected}([2, 1], [3, 1]), \ldots,$
$\texttt{connected}([4, 3], [4, 4]), \texttt{at}(\texttt{agent}, [1, 1]), \texttt{at}(\texttt{wumpus}, [1, 3]),$
$\texttt{at}(\texttt{pit}, [3, 1]), \texttt{at}(\texttt{pit}, [4, 4]), \texttt{arrowleft}, \texttt{agent\_alive}\}$

Operators $\mathcal{O}$:

$\texttt{Move}(x, y)$
  PRE :$\texttt{at}(\texttt{agent}, x) \wedge \texttt{connected}(x, y) \wedge \texttt{agent\_alive}$
  EFF :$\texttt{at}(\texttt{wumpus}, y) \rhd \neg\texttt{agent\_alive},$
    $\texttt{at}(\texttt{pit}, y) \rhd \neg\texttt{agent\_alive},$
    $\texttt{at}(\texttt{agent}, y),$
    $\neg\texttt{at}(\texttt{agent}, x)$

$\texttt{Shoot}(x, y)$
  PRE :$\texttt{at}(\texttt{agent}, x) \wedge \texttt{connected}(x, y) \wedge \texttt{arrowleft} \wedge \texttt{agent\_alive}$
  EFF :$\texttt{at}(\texttt{wumpus}, y) \rhd \texttt{scream},$
    $\neg\texttt{arrowleft}$

Goal $\mathcal{G}$:

$\texttt{scream} \wedge \texttt{agent\_alive}$

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[1] *stench,breeze* and *gold* will not be formalized here and serve only for the purpose of illustration (or confusion?).

(a) Suppose, you want to solve a simplified, monotonic planning problem by ignoring negative effects (aka. the "delete relaxation") in order to calculate a heuristic.
Specify the operators of the relaxed planning task.

(b) Sketch the first two levels of the relaxed planning graph. Facts that do not change in the relaxed problem, e.g. `agent_alive`, $at(pit, x)$ and $connected(x, y)$ can be omitted (In the initial state in layer $F0$ you only have to sketch the fact $at(agent, [1, 1])$).
To further simplify the problem, you may compile away the conditional effect $at(wumpus, y) \triangleright scream$ of $Shoot(x, y)$ by moving the effect precondition to the operator precondition[2].

(c) Contrary to the PlanGraph method presented in the lecture, actions cannot be conflicting in a relaxed planning problem since they neither contain negative preconditions nor negative effects. Therefore, relaxed plans can be found more easily and thus be used to derive heuristic estimates.
Specify the relaxed plan. Is this plan also applicable in the original problem?

Exercise 4.4 (Allen's Interval Calculus)

(a) In general, the composition of two binary relations R and S (over X) is defined as

$$R \circ S = \{(x, z) \mid \exists y \in X \text{ such that } (x, y) \in R \text{ and } (y, z) \in S\}.$$
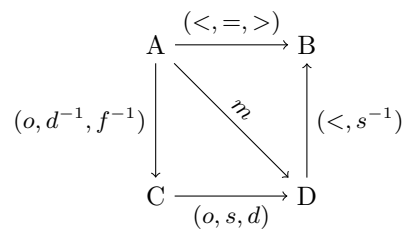
Allen's interval calculus is *closed under composition* which means that every composition of Allen relations (also for unions of the 13 base relations) can be represented as union of base relations. For example, $f \circ s = d$ because for arbitrary intervals A, B and C with AfB and BsC it must hold that AdC. Note that in general the composition of two base relations needs not to result in a single base relation, as you can see from the example $f^{-1} \circ d = (o, d, s)$. Determine the following compositions:

(1) $o \circ m$

(2) $m \circ f$

(3) $(o, m) \circ f$

(b) The composition is also used for the constraint propagation technique. Use this technique to make the following constraint network 3-consistent.

Hint: If there is no directed edge from one interval to another one, this implicitly implies that the all-relation (the union of all 13 base relations) holds for these intervals. You may use in your solution that $(IRJ, ISJ)^{-1} = (IR^{-1}J, IS^{-1}J)$ (for intervals I, J and R, S base relations of Allen's interval calculus).

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[2]When compiling away conditional effects, usually two operators (one with the effect condition and one with the negated effect condition) are created. However, $Shoot'(x, y) = \langle PRE : at(agent, x), \neg at(wumpus, y), \dots EFF : \emptyset \rangle$ does not have any effect and might be excluded here as a result.

$$
\begin{array}{ccc}
A & \xrightarrow{\ (<,=,>)\ } & B \\
\ \downarrow{\scriptstyle (o,\,d^{-1},\,f^{-1})} & \searrow{\scriptstyle m} & \uparrow{\scriptstyle (<,\,s^{-1})} \\
C & \xrightarrow[\ (o,\,s,\,d)\ ]{} & D
\end{array}
$$

Exercise 4.5 (Bayesian Rule)

Assume, you are at night in Athens and witness a car accident in which a taxi is involved. 90% of the taxis in Athens are green, all others are blue. You are absolutely sure that the taxi involved in the accident was blue. But tests show that distinguishing between blue and green at darkness is only 75% reliable. If you take this into consideration, what is the probability that the taxi was really blue? (Hint: Distinguish exactly between the statement that a taxi *is* red and the statement that a taxi *appears* to be red.)

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.