Albert-Ludwigs-Universität Freiburg
Lecture: Introduction to Mobile Robotics
Summer term 2015

Institut für Informatik

Prof. Dr. Wolfram Burgard
Dr. Gian Diego Tipaldi
Dr. Barbara Frank

Sheet 2

Topic: Linear Algebra, Locomotion, and Sensing

Submission deadline: May 7, 2015

Submit to: `mobilerobotics@informatik.uni-freiburg.de`

Transformations between coordinate frames play an important role in robotics. As background for exercises 1 and 3 on this sheet, please refer to the linear algebra slides on affine transformations and transformation combination.

Exercise 1: 2D Transformations as Affine Matrices

The 2D pose of a robot w.r.t. a global coordinate frame is commonly written as $\mathbf{x} = (x, y, \theta)^T$, where (x, y) denotes its position in the xy-plane and θ its orientation. The homogeneous transformation matrix that represents a pose $\mathbf{x} = (x, y, \theta)^T$ w.r.t. to the origin $(0, 0, 0)^T$ of the global coordinate system is given by

$$T = \begin{pmatrix} \mathbf{R}(\theta) & \mathbf{t} \\ 0 & 1 \end{pmatrix}, \mathbf{R}(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}, \mathbf{t} = \begin{pmatrix} x \\ y \end{pmatrix}$$

(a) While being at pose $\mathbf{x_1} = (x_1, y_1, \theta_1)^T$, the robot senses a landmark l at position (l_x, l_y) w.r.t. to its local frame. Use the matrix T_1 to calculate the coordinates of l w.r.t. the global frame.

(b) Now imagine that you are given the landmark's coordinates w.r.t. to the global frame. How can you calculate the coordinates that the robot will sense in his local frame?

(c) The robot moves to a new pose $\mathbf{x_2} = (x_2, y_2, \theta_2)^T$ w.r.t. the global frame. Find the transformation matrix T_{12} that represents the new pose w.r.t. to $\mathbf{x_1}$. *Hint:* Write T_{12} as a product of homogeneous transformation matrices.

(d) Again, the robot measures the position of the landmark l. What will be the result, given the coordinates (l_x, l_y) w.r.t. to $\mathbf{x_1}$?

Exercise 2: Locomotion

A robot equipped with a differential drive starts at position $x = 1.0m$, $y = 2.0m$ and with heading $\theta = \frac{\pi}{2}$. It has to move to the position $x = 1.5m$, $y = 2.0m$, $\theta = \frac{\pi}{2}$ (all angles in radians). The movement of the vehicle is described by steering commands (v_l = speed of left wheel, v_r = speed of right wheel, t = driving time).

(a) What is the minimal number of steering commands (v_l, v_r, t) needed to guide the vehicle to the desired target location?

(b) What is the length of the shortest trajectory under this constraint?

(c) Which sequence of steering commands guides the robot on the shortest trajectory to the desired location if an arbitrary number of steering commands can be used?

(d) What is the length of this trajectory?

Note: the length of a trajectory refers to the travelled distance along the trajectory.

Exercise 3: Sensing

A robot is located at $x = 1.0m$, $y = 0.5m$, $\theta = \frac{\pi}{4}$. Its laser range finder is mounted on the robot at $x = 0.2m$, $y = 0.0m$, $\theta = \pi$ (with respect to the robot's frame of reference).

The distance measurements of one laser scan can be found in the file `laserscan.dat`, which is provided on the website of this lecture. The first distance measurement is taken in the angle $\alpha = -\frac{\pi}{2}$ (in the frame of reference of the laser range finder), the last distance measurement has $\alpha = \frac{\pi}{2}$ (i.e., the field of view of the sensor is π), and all neighboring measurements are in equal angular distance (all angles in radians).

Note: You can load the data file and calculate the corresponding angles in Octave using

```
scan = load("-ascii", "laserscan.dat");
angle = linspace(-pi/2, pi/2, size(scan,2));
```

(a) Use Octave to plot all laser end-points in the frame of reference of the laser range finder.

(b) The provided scan exhibits an unexpected property. Identify it an suggest an explanation.

(c) Use homogeneous transformation matrices in Octave to compute and plot the center of the robot, the center of the laser range finder, and all laser end-points in world coordinates.

Note: You can equally scale the x and y-axis of a plot using

```
axis("equal");
```