Introduction to Mobile Robotics

Wheeled Locomotion

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Locomotion of Wheeled Robots

Locomotion (Oxford Dict.): Power of motion from place to place

- Differential drive (AmigoBot, Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive (B21)
- XR4000
- Mecanum wheels

we also allow wheels to rotate around the z axis
Instantaneous Center of Curvature

- For rolling motion to occur, each wheel has to move along its y-axis
Differential Drive

\[ ICC = [x - R \sin \theta, y + R \cos \theta] \]

\[ \omega(R + l/2) = v_r \]
\[ \omega(R - l/2) = v_l \]
\[ R = \frac{l(v_l + v_r)}{2(v_r - v_l)} \]
\[ \omega = \frac{v_r - v_l}{l} \]
\[ v = \frac{v_r + v_l}{2} \]
Differential Drive: Forward Kinematics

\[
\begin{bmatrix}
    x' \\
    y' \\
    \theta'
\end{bmatrix} =
\begin{bmatrix}
    \cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\
    \sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x - ICC_x \\
    y - ICC_y \\
    \theta
\end{bmatrix} +
\begin{bmatrix}
    ICC_x \\
    ICC_y \\
    \omega \delta t
\end{bmatrix}
\]

\[
x(t) = \int_{0}^{t} v(t') \cos[\theta(t')] \, dt'
\]

\[
y(t) = \int_{0}^{t} v(t') \sin[\theta(t')] \, dt'
\]

\[
\theta(t) = \int_{0}^{t} \omega(t') \, dt'
\]
Differential Drive: Forward Kinematics

\[
\begin{bmatrix}
 x' \\
 y' \\
 \theta'
\end{bmatrix}
= 
\begin{bmatrix}
 \cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\
 \sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\
 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
 x - \text{ICC}_x \\
 y - \text{ICC}_y \\
 \theta
\end{bmatrix}
+ 
\begin{bmatrix}
 \text{ICC}_x \\
 \text{ICC}_y \\
 \omega \delta t
\end{bmatrix}
\]

\[
x(t) = \frac{1}{2} \int_{0}^{t} [v_r(t') + v_l(t')] \cos[\theta(t')] \, dt'
\]

\[
y(t) = \frac{1}{2} \int_{0}^{t} [v_r(t') + v_l(t')] \sin[\theta(t')] \, dt'
\]

\[
\theta(t) = \frac{1}{l} \int_{0}^{t} [v_r(t') - v_l(t')] \, dt'
\]
Ackermann Drive

\[
\text{ICC} = [x - R \sin \theta, y + R \cos \theta]
\]

\[
R = \frac{d}{\tan \varphi}
\]

\[
\omega (R + l/2) = v_r
\]

\[
\omega (R - l/2) = v_l
\]

\[
R = \frac{l (v_l + v_r)}{2 (v_r - v_l)}
\]

\[
\omega = \frac{v_r - v_l}{l}
\]
Synchronous Drive

\[
x(t) = \int_{0}^{t} v(t') \cos[ \theta(t')] dt'
\]

\[
y(t) = \int_{0}^{t} v(t') \sin[ \theta(t')] dt'
\]

\[
\theta(t) = \int_{0}^{t} \omega(t') dt'
\]
XR4000 Drive

\[
x(t) = \int_{0}^{t} v(t') \cos[\theta(t')] dt'
\]

\[
y(t) = \int_{0}^{t} v(t') \sin[\theta(t')] dt'
\]

\[
\theta(t) = \int_{0}^{t} \omega(t') dt'
\]
Mecanum Wheels

\[ v_y = \left( v_0 + v_1 + v_2 + v_3 \right) / 4 \]
\[ v_x = \left( v_0 - v_1 + v_2 - v_3 \right) / 4 \]
\[ v_\theta = \left( v_0 + v_1 - v_2 - v_3 \right) / 4 \]
\[ v_{\text{error}} = \left( v_0 - v_1 - v_2 + v_3 \right) / 4 \]
The Kuka OmniRob Platform
Example: KUKA youBot
Tracked Vehicles
Other Robots: OmniTread

[courtesy by Johann Borenstein]
Non-Holonomic Constraints

- Non-holonomic constraints limit the possible incremental movements within the configuration space of the robot.
- Robots with differential drive or synchro-drive move on a circular trajectory and cannot move sideways.
- XR-4000 or Mecanum-wheeled robots can move sideways (they have no non-holonomic constraints).
Holonomic vs. Non-Holonomic

- Non-holonomic constraints reduce the control space with respect to the current configuration
  - E.g., moving sideways is impossible.

- Holonomic constraints reduce the configuration space.
  - E.g., a train on tracks (not all positions and orientations are possible)
Drives with Non-Holonomic Constraints

- Synchro-drive
- Differential drive
- Ackermann drive
Drives without Non-Holonomic Constraints

- XR4000 drive
- Mecanum wheels
Dead Reckoning and Odometry

- Estimating the motion based on the issued controls/wheel encoder readings
- Integrated over time
Summary

- Introduced different types of drives for wheeled robots
- Math to describe the motion of the basic drives given the speed of the wheels
- Non-holonomic constraints
- Odometry and dead reckoning