

# Introduction to Mobile Robotics

## Multi-Robot Exploration

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# Exploration

- The approaches seen so far are purely passive
- Given an unknown environment, how can we control multiple robots to efficiently learn a map?
- By reasoning about control, the mapping process can be made much more effective

# Decision-Theoretic Formulation of Exploration

$$\pi(Bel) =$$

$$\operatorname{argmax}_u \left[ E_z [I_{Bel}(z, u)] - \alpha \int_x r(x, u) Bel(x) dx \right]$$

reward

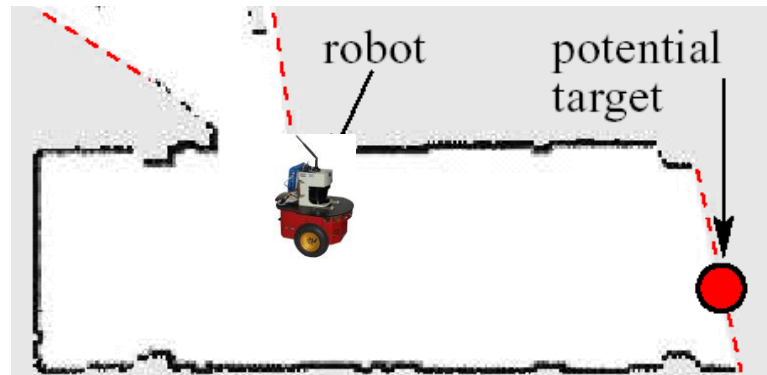
(expected information gain)

cost

(path length)

# Single Robot Exploration

- Frontiers between free space and unknown areas are potential target locations
- Going to frontiers will gain information



- Select the target that minimizes a cost function (e.g. travel time / distance /...)

# Multiple Robots

Multiple robots: how to control them to optimize the performance of the whole team?

- Exploration
- Path planning
- Action planning ...

# Exploration: The Problem

## Given:

- Unknown environment
- Team of robots

## Task:

- Coordinate the robots to efficiently learn a complete map of the environment

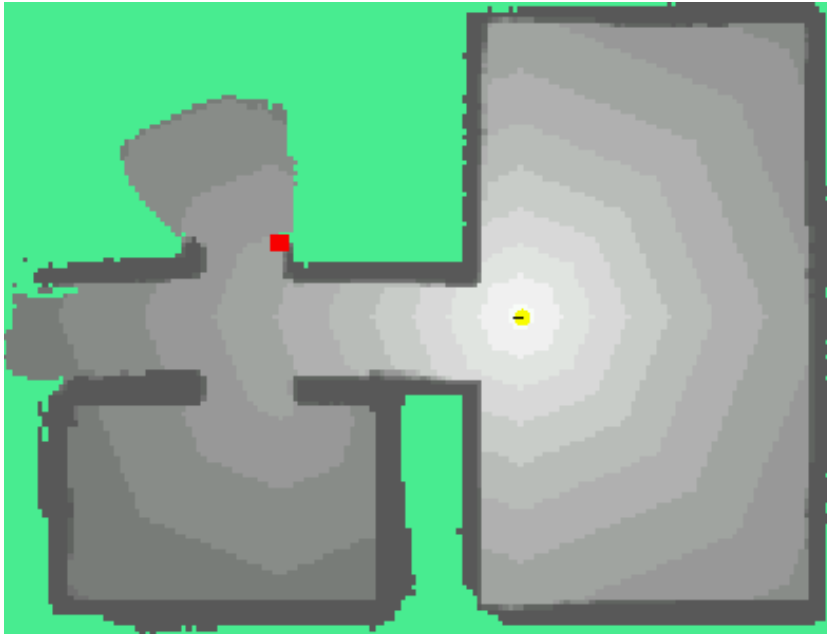
## Complexity:

- Exponential in the number of robots

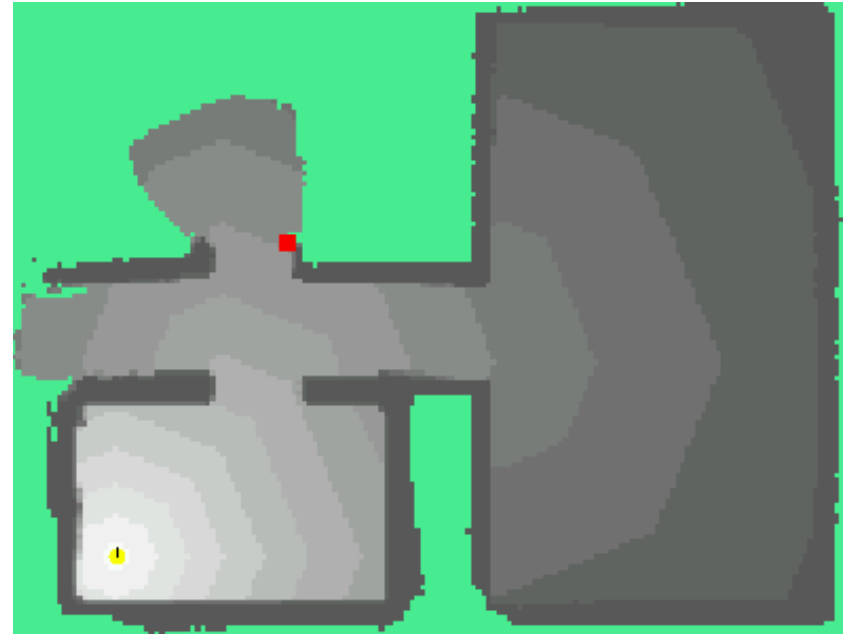


# Example

**Robot 1:**



**Robot 2:**



# Levels of Coordination

- No exchange of information
- **Implicit coordination (uncoordinated):**  
Sharing a joint map [Yamauchi et.al, 98]
  - Communication of the individual maps and poses
  - Central mapping system
- **Explicit coordination:** Improve assignment of robots to target points
  - Communication of the individual maps and poses
  - Central mapping system
  - Central planner for target point assignment



# Realizing Explicit Coordination for Multi-Robot Exploration

- Robots share a common map
- Frontiers between free space and unknown areas are potential target locations
- Find a good assignment of frontier locations to robots to minimize exploration time and maximize information gain

# Key Ideas

1. Choose target locations at the **frontier** to the **unexplored area** by trading off the **expected information gain** and **travel costs**.
2. **Reduce utility** of target locations whenever they are expected to be **covered by another robot**.
3. Use **on-line mapping and localization** to compute the joint map.

# The Coordination Algorithm (informal)

1. Determine the frontier cells.
2. Compute for each robot the cost for reaching each frontier cell.
3. Choose the robot with the optimal overall evaluation and assign the corresponding target point to it.
4. Reduce the utility of the frontier cells visible from that target point.
5. If there is one robot left go to 3.

# The Coordination Algorithm

1. Determine the set of frontier cells
2. Compute for each robot  $i$  the cost  $V_{x,y}^i$  for reaching each frontier cell
3. Set the utility  $U_{x,y}$  of all frontier cells to 1
4. While there is one robot left without a target point
  - (a) Determine a robot  $i$  and a frontier cell  $\langle x, y \rangle$  which satisfy

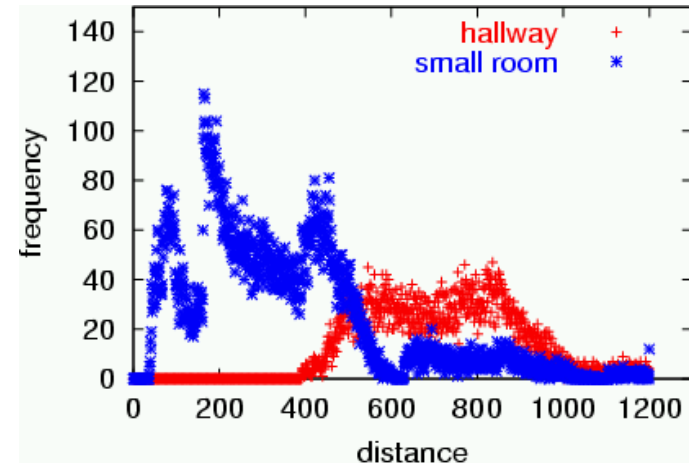
$$(i, \langle x, y \rangle) = \operatorname{argmax}_{(i', \langle x', y' \rangle)} U_{x', y'} - V_{x', y'}^{i'}$$

- (b) Reduce the utility of each target point  $\langle x', y' \rangle$  in the visibility area according to

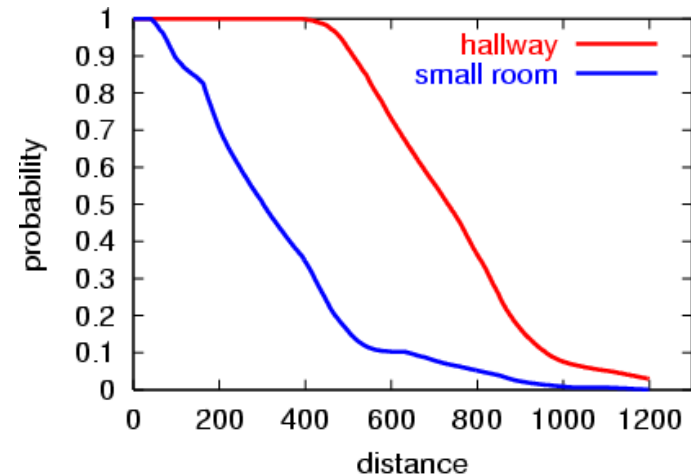
$$U_{x', y'} \leftarrow U_{x', y'} \cdot (1 - P(\| \langle x, y \rangle - \langle x', y' \rangle \|))$$

# Estimating the Visible Area

Distances measured during exploration:

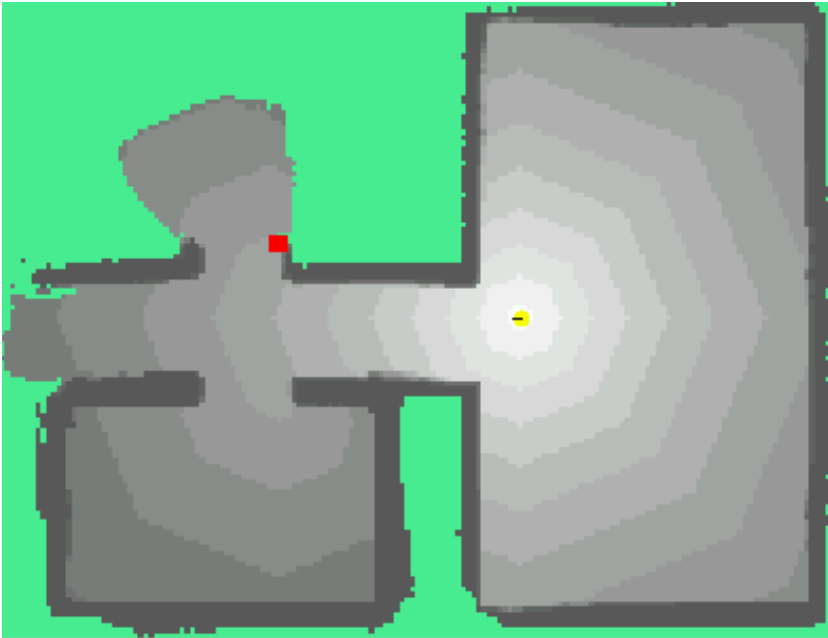


Resulting probability of measuring at least distance  $d$ :

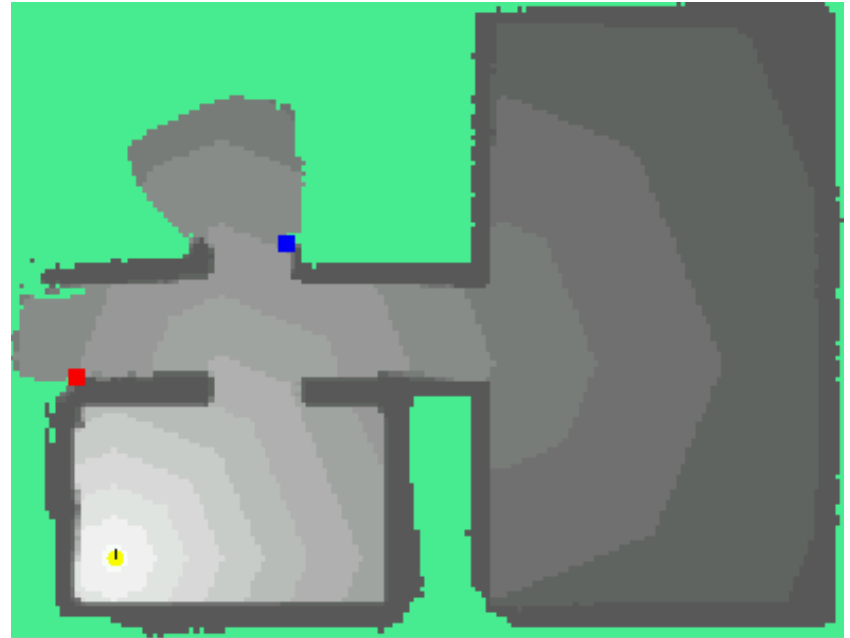


# Application Example

First robot:



Second robot:

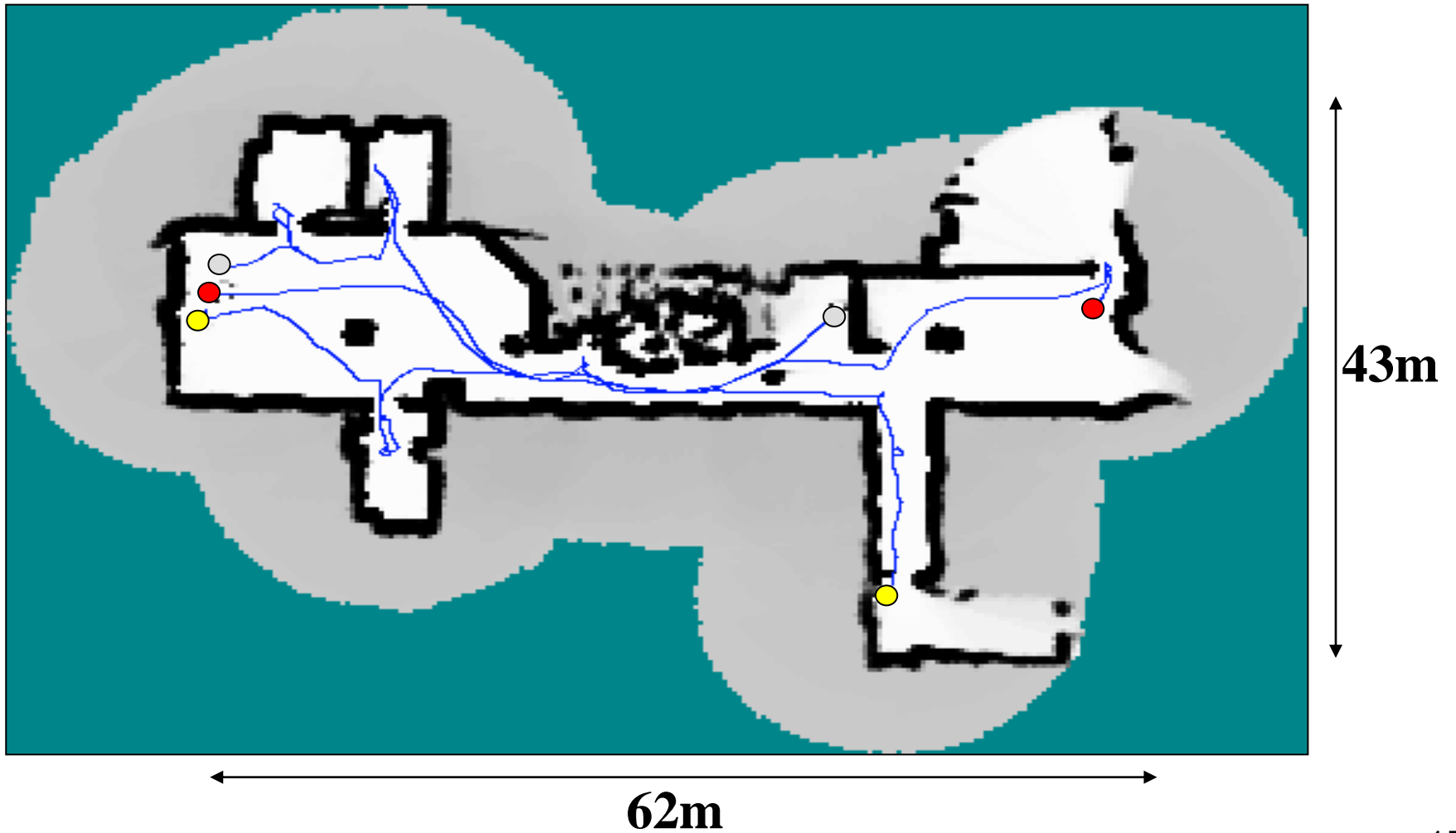


# Multi-Robot Exploration and Mapping of Large Environments

Multi-Robot Mapping  
and Exploration

Carnegie Mellon  
October 1999

# Resulting Map

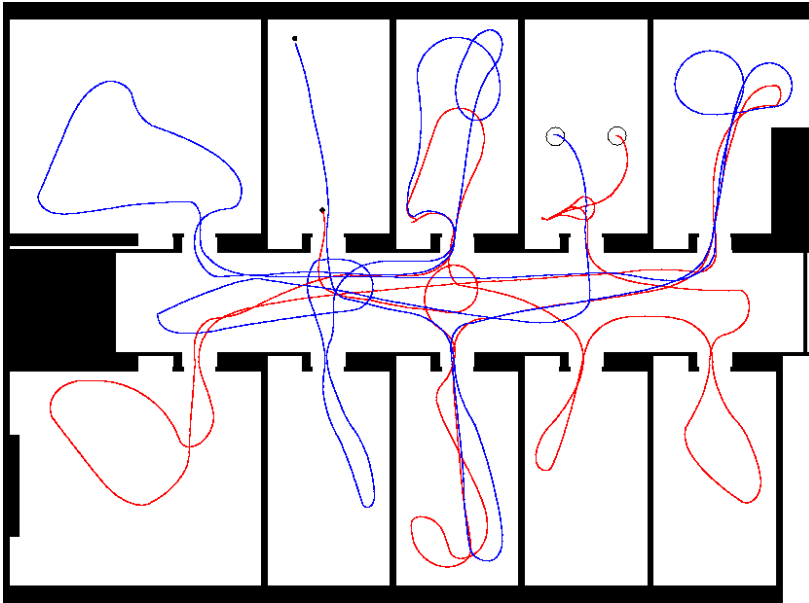




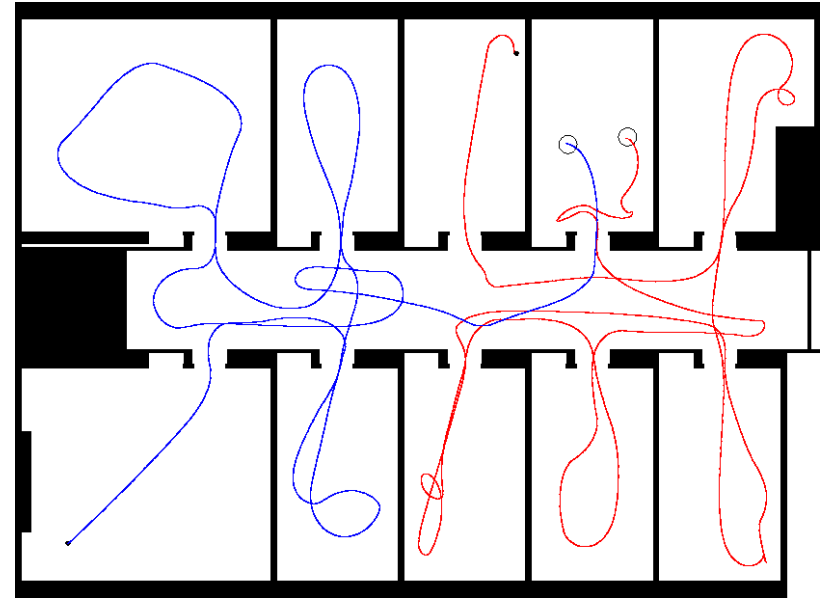


# Typical Trajectories in an Office Environment

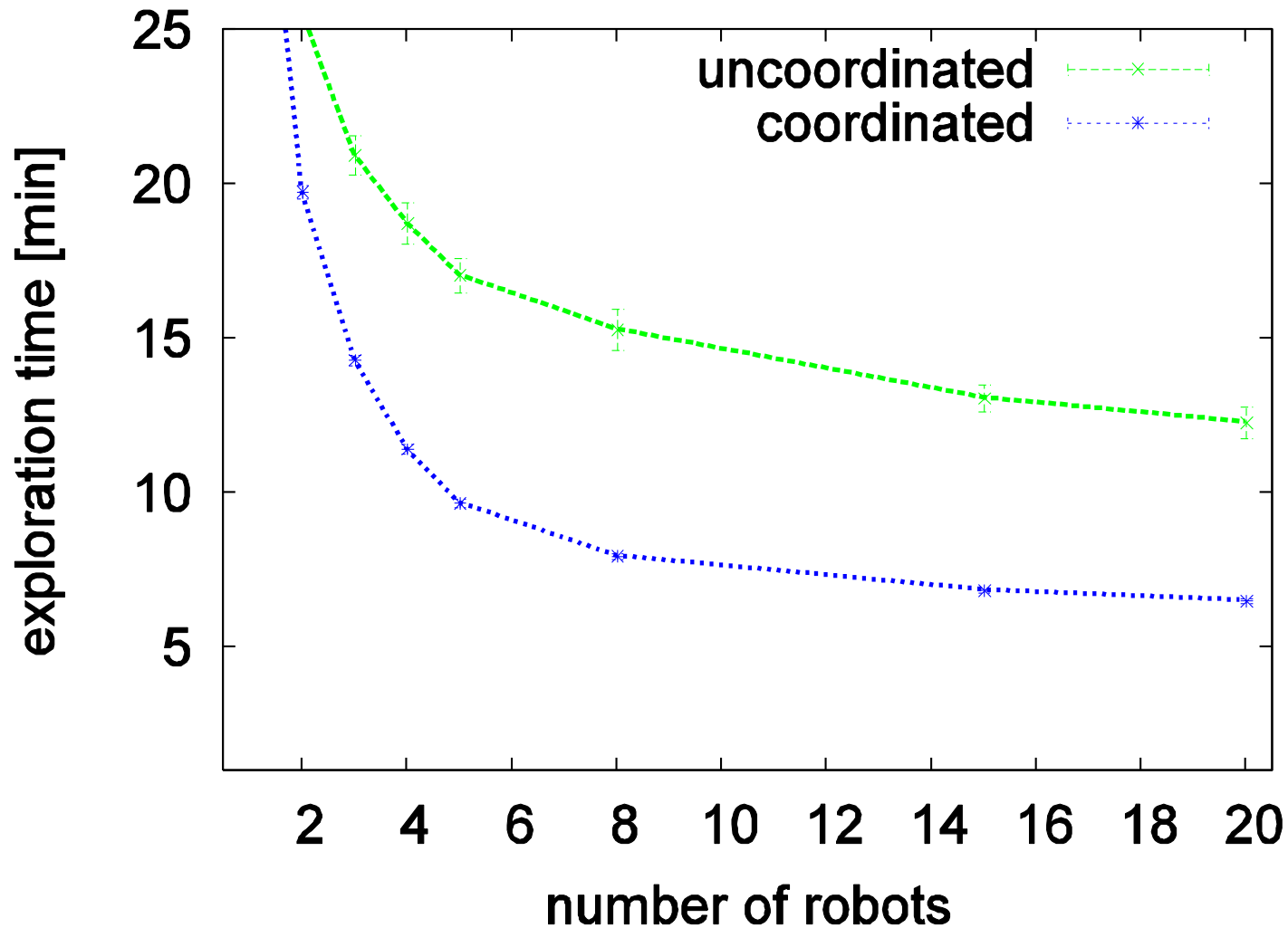
Implicit coordination:



Explicit coordination:

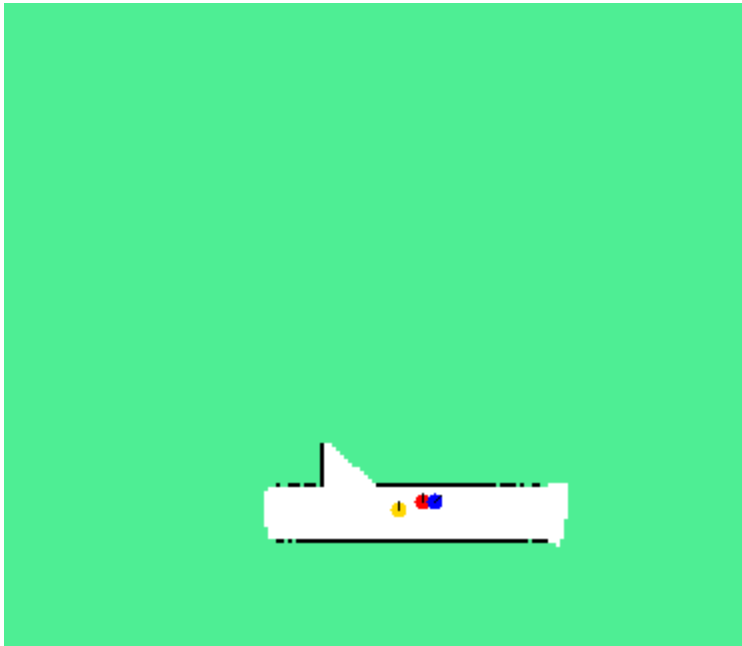


# Exploration Time

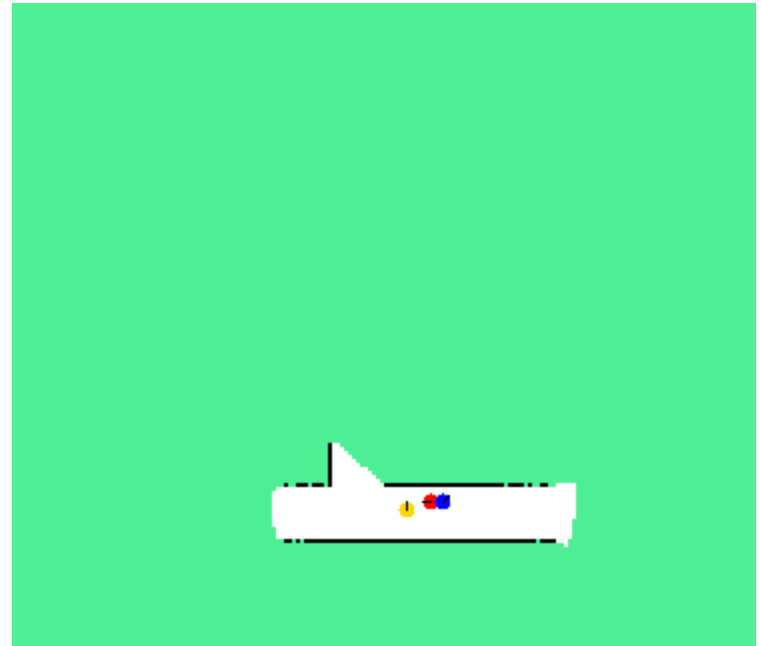


# Simulation Experiments

**Implicitly coordinated:**

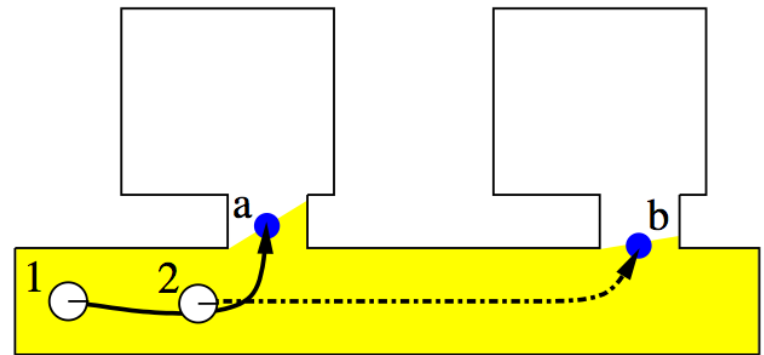
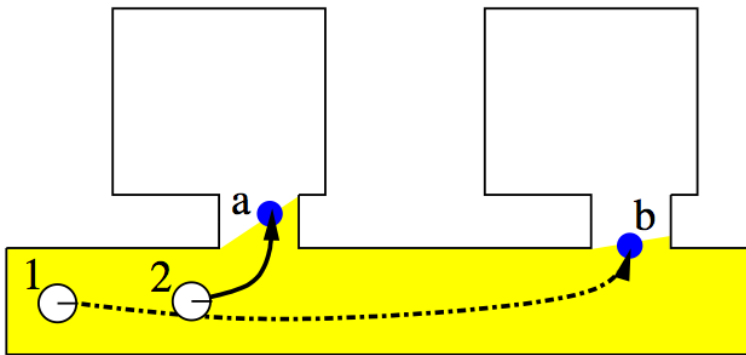


**Explicitly coordinated:**



# Optimizing Assignments

- The current system performs a greedy assignment of robots to target locations



- What if we optimize the assignment?

# Optimizing Assignment Algorithm

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**Algorithm 2** Goal selection determining the best assignment over all permutations.

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- 1: Determine the set of frontier cells.
  - 2: Compute for each robot  $i$  the cost  $V_t^i$  for reaching each frontier cell.
  - 3: Determine target locations  $t_1, \dots, t_n$  for the robots  $i = 1, \dots, n$  that maximizes the following evaluation function:  
$$\sum_{i=1}^n U(t_i \mid t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_n) - \beta \cdot (V_{t_i}^i)^2.$$
- 

One approach: randomized optimization of assignments.

# General Idea for Optimization

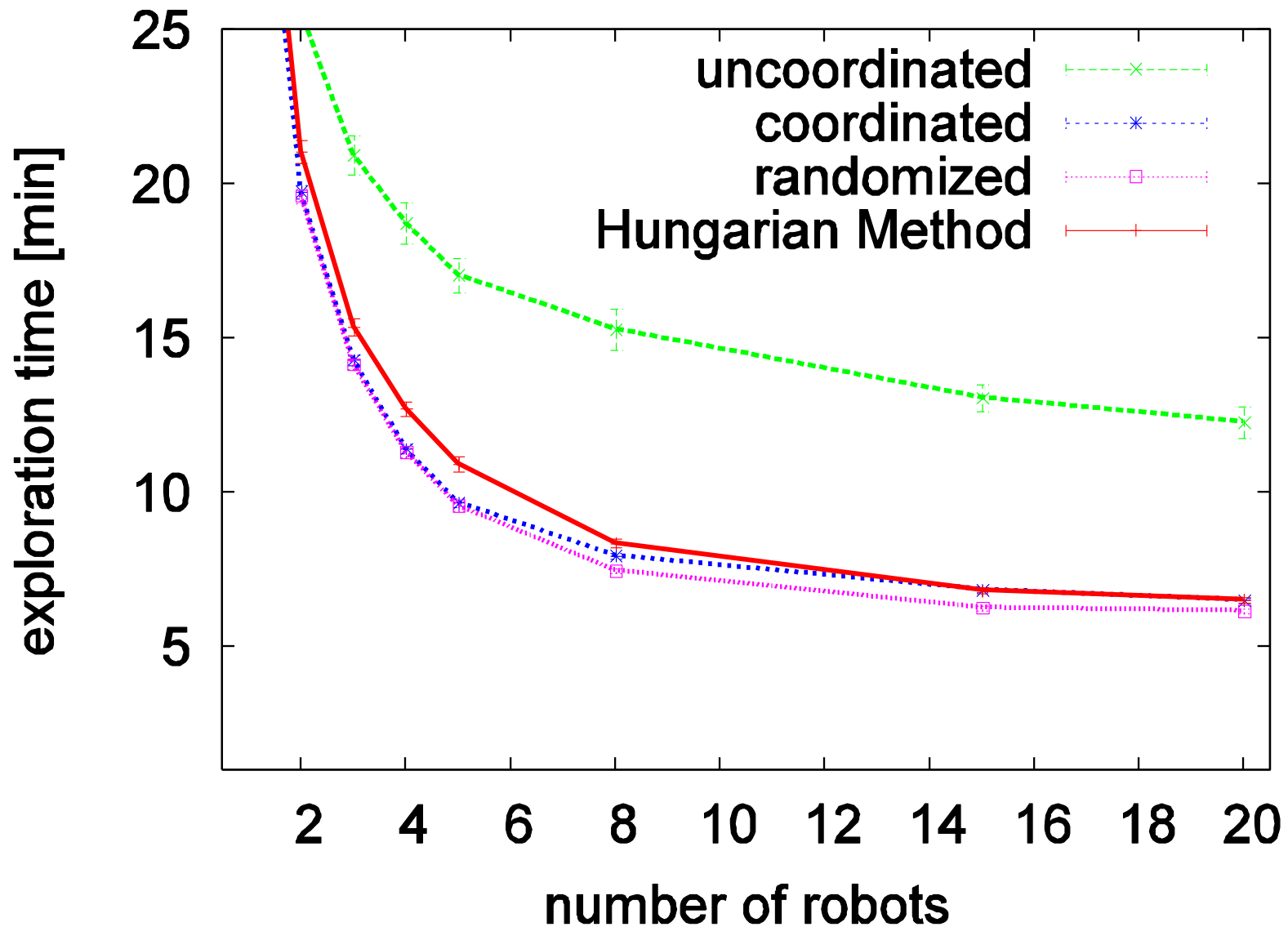
1. Start with an initial assignment
  2. Select a pair of robot/target point assignments
  3. If the evaluation improves we swap the assignments
- Variants:
    - Accept lower evaluations with a certain but over time decreasing probability
    - Perform random restarts

# Other Coordination Techniques

- Hungarian Method:
  - Optimal assignment of jobs to machines given a fixed cost matrix.
  - Similar results that the presented coordination technique.
- Market economy-guided approaches:
  - Robots trade with targets.
  - Computational load is shared between the robots



# Exploration Time



# Summary on Exploration

- Efficient coordination leads to reduced exploration times
- In general exponential in the team size
- Efficient polynomial approximations
- Distributing the robots over the environment is key to efficiency
- Methods trade off the cost of an action and the expected utility of reaching the corresponding frontier (target location)

# Other Problems

- Unknown starting locations
- Exploration under position uncertainty
- Limited communication abilities
- Efficient exchange of information
- ...