Foundations of Artificial Intelligence

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Exercise Sheet 3

Due: Wednesday, June 1, 2016, before the lecture

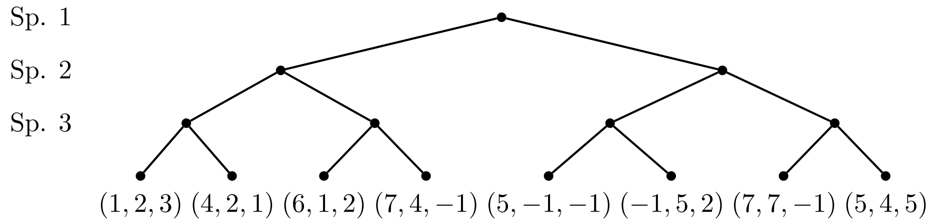
Exercise 3.1

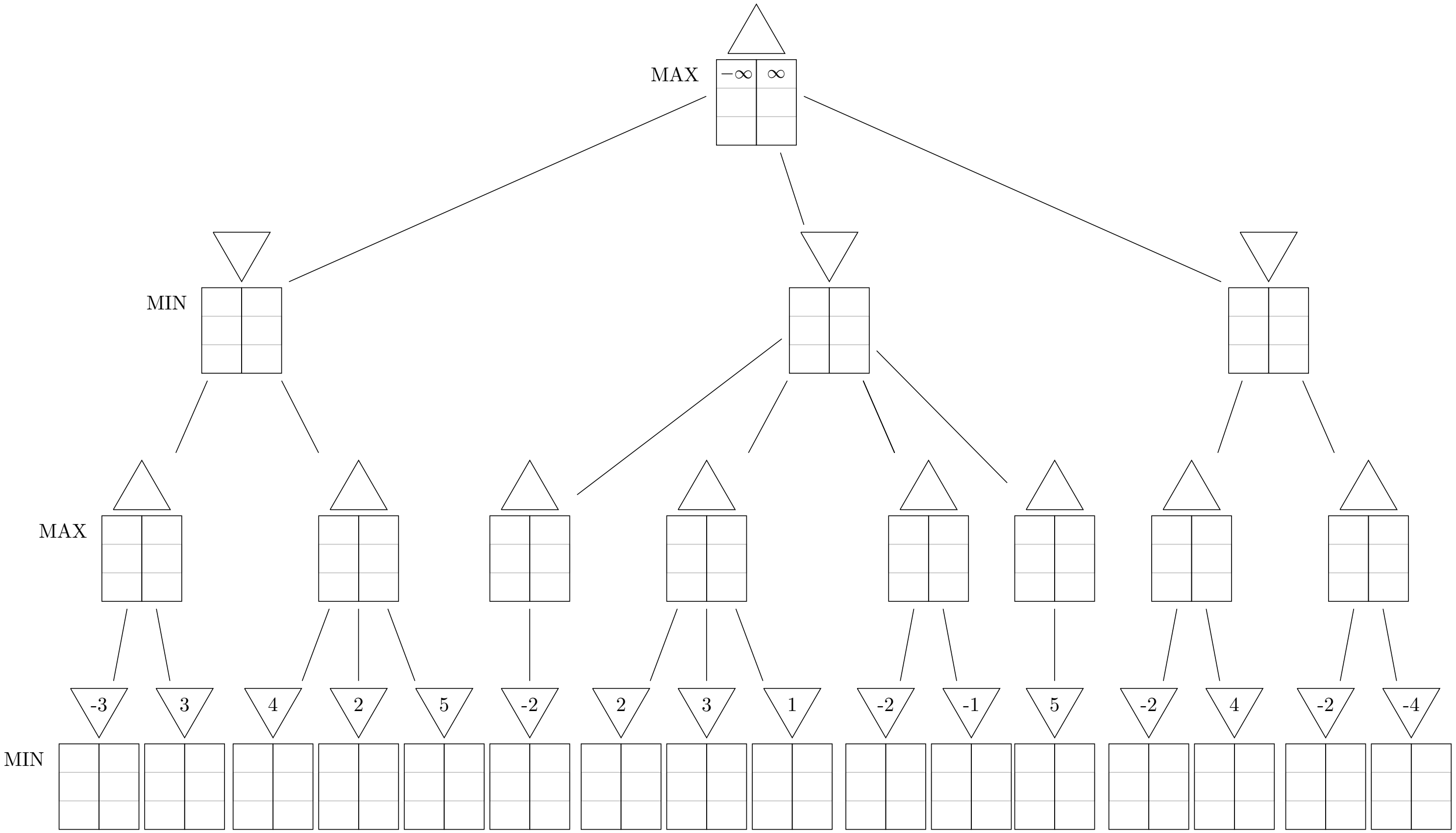
Board Games

(a) Consider the game tree for the two-person board game depicted on the following page.
Simulate the behavior of the Minimax algorithm with α-β pruning (always expand children from left to right). Enter the computed node values into the triangles and all intermediate α-β values into the appropriate tables.

(b) Consider the problem of search in a three-player game (you may assume that no alliances are allowed) without the zero-sum condition. The players are called 1, 2, and 3. Unlike in the case of two-player zero-sum games, the evaluation function now returns a triple (x_1, x_2, x_3) such that x_i is the value the node has for player i.

Complete the game tree given below by annotating all interior nodes and the root node with the backed-up value triples.

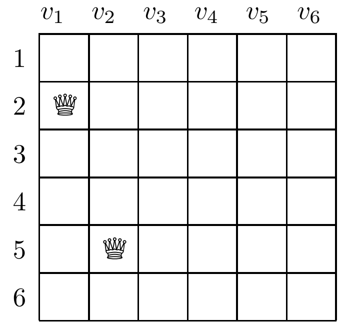




(the minimax tree for question (1)

Exercise 3.2 (Forward Checking / Arc consistency)

Consider the 6-queens problem, where 6 pieces have to be placed on a size 6×6 board in such a way that no two queens are on the same horizontal, vertical or diagonal line. Let the domains be $dom(v_i) = 1, \ldots, 6$ for all variables $v_i \in V$. Consider now state $\alpha = \{v_1 \mapsto 2, v_2 \mapsto 5\}$.



(a) Enforce arc consistency in α. Specify in particular the domains of the variables before and after applying arc consistency. You may assume that the domain of variables with allocated values only consists of that value, while the values of unassigned variables still range over the complete domain. Always choose the variable with lowest index, for which arc consistency has not been established yet.

(b) Apply forward-checking in α. Compare with the result of (a).

Exercise 3.3 (Satisfiability, Models)

(a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.

(1) $\mathit{Smoke} \Rightarrow \mathit{Smoke}$

(2) $\mathit{Smoke} \Rightarrow \mathit{Fire}$

(3) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow (\neg \mathit{Fire} \Rightarrow \neg \mathit{Smoke})$

(4) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow ((\mathit{Smoke} \wedge \mathit{Heat}) \Rightarrow \mathit{Fire})$

(5) $\mathit{TheBestTeamWins} \Leftrightarrow \mathit{GermanyWinsEuropeanChampionship}$

(b) Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following formulae? Explain.

(1) $(A \wedge B) \vee (B \wedge C)$

(2) $A \vee B$

(3) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 3.4 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ, and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \tag{1}$$
$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \tag{2}$$
$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \tag{3}$$
$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \tag{4}$$
$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \tag{5}$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.

Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show whether $K \models (\neg B \rightarrow (A \wedge C))$ holds.

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.