Foundations of Artificial Intelligence

Dr. J. Boedecker, Prof. Dr. W. Burgard, PD Dr. M. Ragni — University of Freiburg
J. Aldinger, J. Boedecker, C. Dornhege, M. Krawez — Department of Computer Science
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Exercise Sheet 4

Due: Wednesday, June 15, 2016, before the lecture

Exercise 4.1 (DPLL)

Use the Davis-Putnam-Logemann-Loveland (DPLL) procedure to find a satisfying assignment for the formula ϕ. Write down all steps carried out by the algorithm during the process. If you have to apply a splitting rule, split on variables in alphabetical order, trying *true* first, then *false*. Indicate the satisfying assignment.

$\phi = (\neg A \vee C \vee \neg D) \wedge (A \vee B \vee C \vee \neg D) \wedge (\neg A \vee \neg E) \wedge \neg C \wedge (A \vee D) \wedge (A \vee C \vee E) \wedge (D \vee E)$

Exercise 4.2 (Semantics of Predicate Logic)

Consider the Interpretation $\mathcal{I} =< \mathcal{D}, \cdot^{\mathcal{I}} >$ with

- $D = \{0, 1, 2, 3\}$
- $even^{\mathcal{I}} = \{0, 2\}$
- $odd^{\mathcal{I}} = \{1, 3\}$
- $lessThan^{\mathcal{I}} = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)\}$
- $two^{\mathcal{I}} = 2$
- $plus^{\mathcal{I}} : D \times D \to D, plus^{\mathcal{I}}(a, b) = (a + b) \mod 4$

and the variable assignment $\alpha = \{(x, 0), (y, 1)\}$.
Decide for the following formulae θ_i if $\mathcal{I}$ is a model for θ_i under α, i.e. if $\mathcal{I}, \alpha \models \theta_i$. Explain your answer by formally applying the semantics.

(a) $\theta_1 = odd(y) \wedge even(two)$

(b) $\theta_2 = \forall x\ (even(x) \vee odd(x))$

(c) $\theta_3 = \forall x \exists y\ lessThan(x, y)$

(d) $\theta_4 = \forall x\ (even(x) \Rightarrow \exists y\ lessThan(x, y))$

(e) $\theta_5 = \forall x\ (odd(x) \Rightarrow even(plus(x, y)))$

Exercise 4.3 (Clausal Normal Form)

Transform the following PL1 formula in Clausal Normal Form.

(a) $\exists x\, [\forall y\, (P(x, y) \Rightarrow \neg Q(f(x))) \vee \exists y \neg\, (\neg P(f(y), x) \vee Q(x))]$

(b) $\forall x \big[\exists y \forall z\, (R(x) \vee S(y, z)) \iff \neg \forall w S(w, x)\big]$

(c) $\forall y \big[\forall x P(x, y) \Rightarrow \forall x \left(\neg U(x) \Rightarrow T(z) \right) \big]$

Exercise 4.4 (Unification)

Find (if possible) the most general unifier with the algorithm presented in the lecture.

(a) $\{P(x, f(x, y), z), P(a, z, f(x, g(b)))\}$

(b) $\{P(g(x, y), f(a, x), z), P(\tilde{x}, f(\tilde{y}, \tilde{x}), x)\}$

(c) $\{Q(x, f(h(a), y)), Q(y, z), Q(h(\tilde{y}), f(\tilde{x}, h(\tilde{y})))\}$

Exercise 4.5 (Resolution)

Consider the following statements about the set of natural numbers:

i If x is divisible by y, then x is greater than or equal to y.

ii If x is greater than or equal to y and y is greater than or equal to x than x is equal to y.

iii If x is divisible by y and y is divisible by x then x is equal to y.

(a) Formalize the statements (i)-(iii) in PL1 using appropriate predicates.

(b) Use resolution to show, $(i) \wedge (ii) \models (iii)$ holds or not.

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.