Foundations of Artificial Intelligence

4. Informed Search Methods

Heuristics, Local Search Methods, Genetic Algorithms

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Best-First Search

Search procedures differ in the way they determine the next node to expand.

Uninformed Search: Rigid procedure with no knowledge of the cost of a given node to the goal.

Informed Search: Knowledge of the worth of expanding a node n is given in the form of an *evaluation function* $f(n)$, which assigns a real number to each node. Mostly, $f(n)$ includes as a component a *heuristic function* $h(n)$, which estimates the costs of the cheapest path from n to the goal.

Best-First Search: Informed search procedure that expands the node with the "best" f-value first.

General Algorithm

```
function TREE-SEARCH(problem) returns a solution, or failure
  initialize the frontier using the initial state of problem
  loop do
     if the frontier is empty then return failure
     choose a leaf node and remove it from the frontier
     if the node contains a goal state then return the corresponding solution
     expand the chosen node, adding the resulting nodes to the frontier
```

Best-first search is an instance of the general TREE-SEARCH algorithm in which *frontier* is a priority queue ordered by an evaluation function f.

When f is always correct, we do not need to search!

Greedy Search

A possible way to judge the "worth" of a node is to estimate its path-costs to the goal.

$$h(n) = \textit{estimated path-costs from } n \textit{ to the goal}$$

The only real restriction is that $h(n) = 0$ if n is a goal.

A best-first search using $h(n)$ as the evaluation function, i.e., $f(n) = h(n)$ is called a *greedy search*.

Example: route-finding problem:
$h(n) =$ straight-line distance from n to the goal

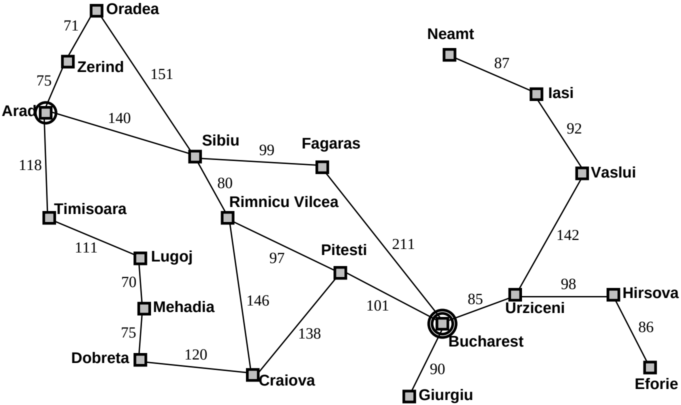
Heuristics

The evaluation function h in greedy searches is also called a *heuristic* function or simply a *heuristic*.

- The word *heuristic* is derived from the Greek word $\varepsilon\upsilon\rho\iota\sigma\kappa\varepsilon\iota\nu$ (note also: $\varepsilon\upsilon\rho\eta\kappa\alpha$!)
- The mathematician Polya introduced the word in the context of problem solving techniques.
- In AI it has two meanings:
  - Heuristics are fast but in certain situations incomplete methods for problem-solving [Newell, Shaw, Simon 1963] (The greedy search is actually generally incomplete).
  - Heuristics are methods that improve the search in the average-case.

$\rightarrow$ In all cases, the heuristic is *problem-specific* and *focuses* the search!

Greedy Search Example



Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
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Sibiu	253
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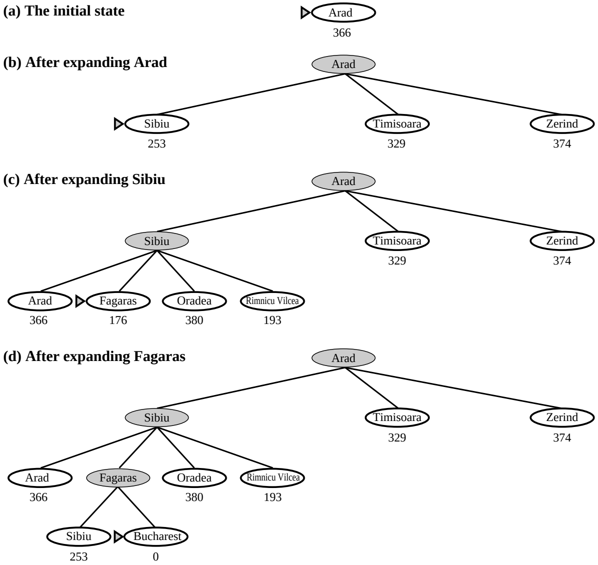
Vaslui	199
Zerind	374

Greedy Search from Arad to Bucharest



Greedy Search - Properties

- a good heuristic might reduce search time drastically
- non-optimal
- incomplete
- graph-search version is complete only in finite spaces

Can we do better?

A^*: Minimization of the Estimated Path Costs

A^* combines the greedy search with the uniform-search strategy:
Always expand node with lowest $f(n)$ first, where

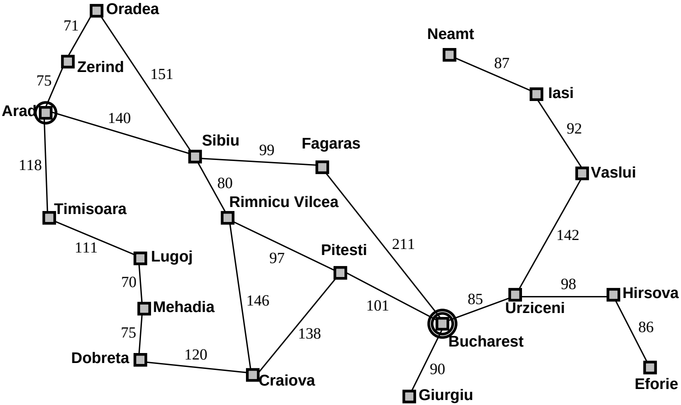
$g(n) =$ actual cost from the initial state to n.
$h(n) =$ estimated cost from n to the next goal.
$f(n) = g(n) + h(n)$,
the estimated cost of the cheapest solution through n.

Let $h^*(n)$ be the actual cost of the optimal path from n to the next goal.
h is *admissible* if the following holds for all n:

$$h(n) \leq h^*(n)$$

We require that for A^*, h is admissible (example: straight-line distance is admissible).
In other words, h is an *optimistic* estimate of the costs that actually occur.

A* Search Example



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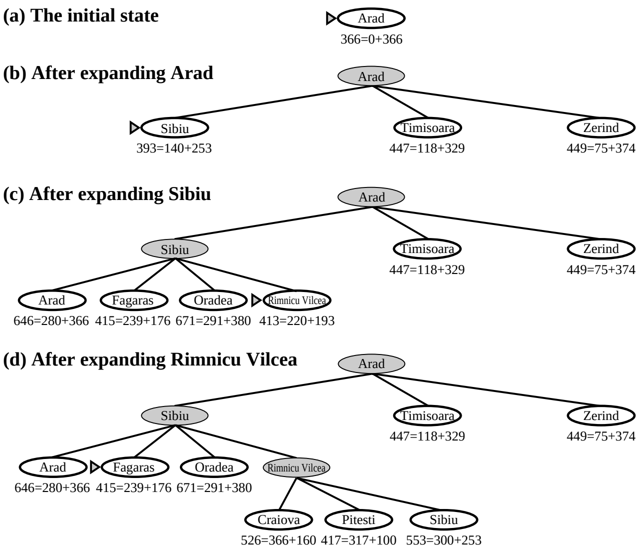
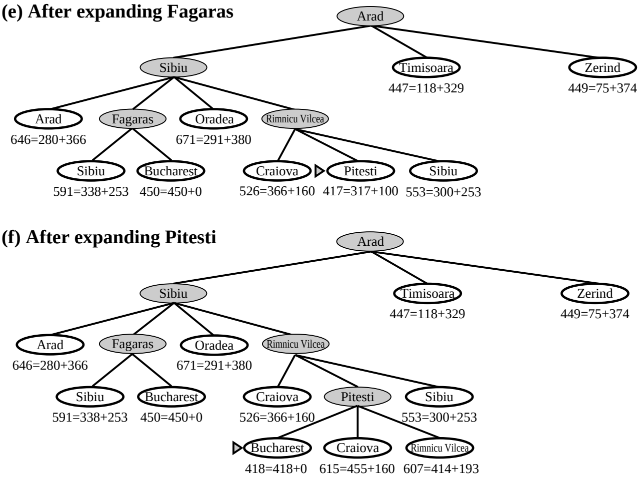
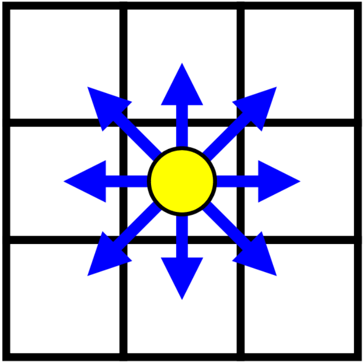
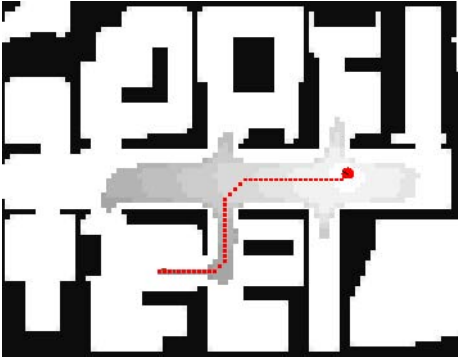
A* Search from Arad to Bucharest



A* Search from Arad to Bucharest



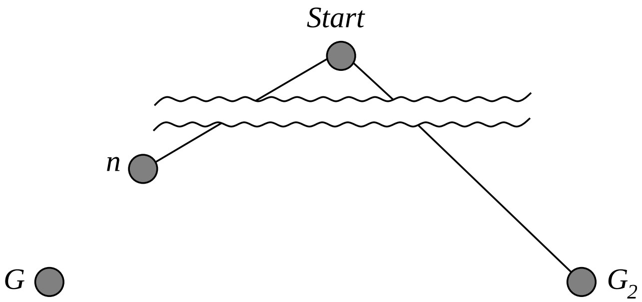




Optimality of A*

Claim: The first solution found has the minimum path cost.

Proof: Suppose there exists a goal node G with optimal path cost f^*, but A* has found another node G_2 with $g(G_2) > f^*$.



Optimality of A*

Let n be a node on the path from the start to G that has not yet been expanded. Since h is admissible, we have

$$f(n) \leq f^*.$$

Since n was not expanded before G_2, the following must hold:

$$f(G_2) \leq f(n)$$

and

$$f(G_2) \leq f^*.$$

It follows from $h(G_2) = 0$ that

$$g(G_2) \leq f^*.$$

$\rightarrow$ Contradicts the assumption!

Completeness and Complexity

Completeness:

If a solution exists, A* will find it provided that (1) every node has a finite number of successor nodes, and (2) there exists a positive constant $\delta > 0$ such that every step has at least cost δ.

$\rightarrow$ there exists only a finite number of nodes n with $f(n) \leq f^*$.

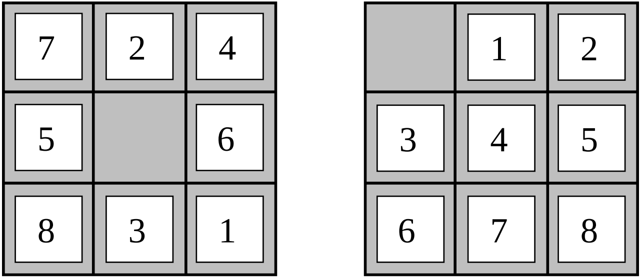
Complexity:

In general, still exponential in the path length of the solution (space, time)

More refined complexity results depend on the assumptions made, e.g. on the quality of the heuristic function. Example:

In the case in which $|h^*(n) - h(n)| \leq O(\log(h^*(n)))$, only one goal state exists, and the search graph is a tree, a sub-exponential number of nodes will be expanded [Gaschnig, 1977, Helmert & Roeger, 2008].

Heuristic Function Example



Start State Goal State

h_1 = the number of tiles in the wrong position

h_2 = the sum of the distances of the tiles from their goal positions (*Manhattan distance*)

Empirical Evaluation

- d = distance from goal
- Average over 100 instances

	Search Cost (nodes generated)			Effective Branching Factor		
d	IDS	A$^*(h_1)$	A$^*(h_2)$	IDS	A$^*(h_1)$	A$^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	-	539	113	-	1.44	1.23
16	-	1301	211	-	1.45	1.25
18	-	3056	363	-	1.46	1.26
20	-	7276	676	-	1.47	1.47
22	-	18094	1219	-	1.48	1.28
24	-	39135	1641	-	1.48	1.26

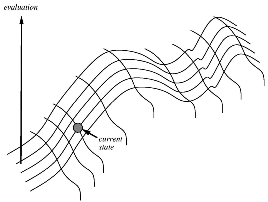
Variants of A*

A* in general still suffers from exponential memory growth. Therefore, several refinements have been suggested:

- iterative-deepening A*, where the f-costs are used to define the cutoff (rather than the depth of the search tree): **IDA***
- Recursive Best First Search (RBFS): introduces a variable f_limit to keep track of the best alternative path available from any ancestor of the current node. If current node exceeds this limit, recursion unwinds back to the alternative path.
- other alternatives memory-bounded A* (MA*) and simplified MA* (SMA*).

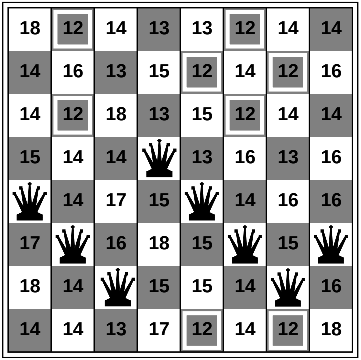
Local Search Methods

- In many problems, it is unimportant how the goal is reached—only the goal itself matters (8-queens problem, VLSI Layout, TSP).
- If in addition a quality measure for states is given, a **local search** can be used to find solutions.
- It operates using a single current node (rather than multiple paths)
- It requires little memory
- Idea: Begin with a randomly-chosen configuration and improve on it step by step $\rightarrow$ **Hill Climbing**.
- Note: It can be used for maximization or minimization respectively (see 8-queens example)



Example: 8-queens Problem (1)

Example state with heuristic cost estimate $h = 17$ (counts the number of pairs threatening each other directly or indirectly).

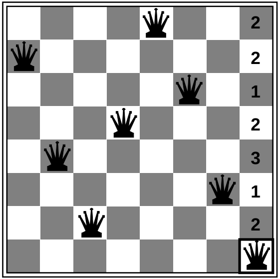


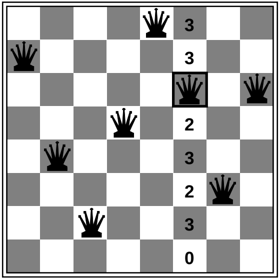
Hill Climbing

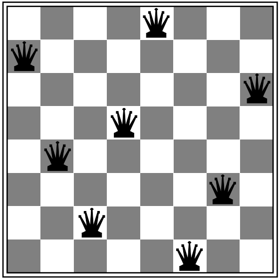
```
function HILL-CLIMBING(problem) returns a state that is a local maximum

  current ← MAKE-NODE(problem.INITIAL-STATE)
  loop do
     neighbor ← a highest-valued successor of current
     if neighbor.VALUE ≤ current.VALUE then return current.STATE
     current ← neighbor
```

Possible realization of a hill-climbing algorithm:
Select a column and move the queen to the square with the fewest conflicts.







Problems with Local Search Methods

- *Local maxima:* The algorithm finds a sub-optimal solution.
- *Plateaus:* Here, the algorithm can only explore at random.
- Ridges: Similar to plateaus but might even require suboptimal moves.

Solutions:

- *Start over* when no progress is being made.
- "Inject noise" $\rightarrow$ random walk

Which strategies (with which parameters) are successful (within a problem class) can usually only empirically be determined.

Example: 8-queens Problem (Local Minimum)

Local minimum ($h = 1$) of the 8-queens Problem. Every successor has a higher cost.

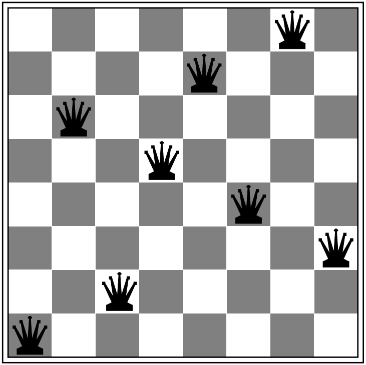
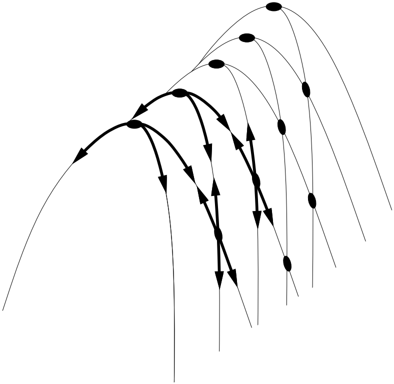


Illustration of the ridge problem

The grid of states (dark circles) is superimposed on a ridge rising from left to right, creating a sequence of local maxima, that are not directly connected to each other. From each local maximum, all the available actions point downhill.



Performance figures for the 8-queens Problem

The 8-queens problem has about $8^8 \approx 17\ million$ states. Starting from a random initialization, hill-climbing directly finds a solution in about 14% of the cases. On average it requires only 4 steps!

Better algorithm: Allow sideways moves (no improvement), but restrict the number of moves (avoid infinite loops!).

E.g.: max. 100 moves: Solves 94%, number of steps raises to 21 steps for successful instances and 64 for failure cases.

Simulated Annealing

In the simulated annealing algorithm, "noise" is injected systematically: first a lot, then gradually less.

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
  inputs: $problem$, a problem
    $schedule$, a mapping from time to "temperature"

  $current \leftarrow$ MAKE-NODE($problem$.INITIAL-STATE)
  for t = 1 **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if T = 0 **then return** $current$
    $next \leftarrow$ a randomly selected successor of $current$
    $\Delta E \leftarrow next$.VALUE – $current$.VALUE
    if $\Delta E > 0$ **then** $current \leftarrow next$
    else $current \leftarrow next$ only with probability $e^{\Delta E/T}$

Has been used since the early 80's for VSLI layout and other optimization problems.

Genetic Algorithms

Evolution appears to be very successful at finding good solutions.

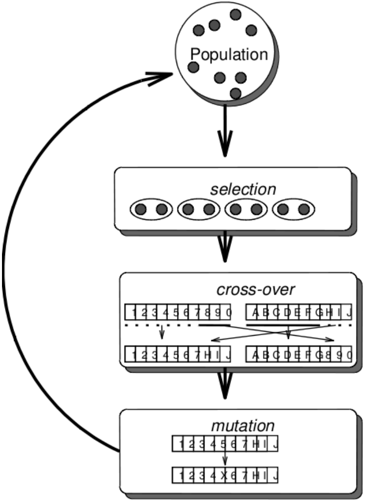
Idea: Similar to evolution, we search for solutions by three operators: "mutation", "crossover", and "selection".

Ingredients:

- Coding of a solution into a string of symbols or bit-string
- A fitness function to judge the worth of configurations
- A population of configurations

Example: 8-queens problem as a chain of eight numbers. Fitness is judged by the number of non-attacks. The population consists of a set of arrangements of queens.

Selection, Mutation, and Crossing



Many variations:

how selection will be applied, what type of cross-over operators will be used, etc.

Selection of individuals according to a fitness function and pairing

Calculation of the breaking points and recombination

According to a given probability elements in the string are modified.

Summary

- Heuristics focus the search
- Best-first search expands the node with the highest worth (defined by any measure) first.
- With the minimization of the evaluated costs to the goal h we obtain a greedy search.
- The minimization of $f(n) = g(n) + h(n)$ combines uniform and greedy searches. When $h(n)$ is admissible, i.e., h^* is never overestimated, we obtain the A* search, which is complete and optimal.
- IDA* is a combination of the iterative-deepening and A* searches.
- Local search methods only ever work on one state, attempting to improve it step-wise.
- Genetic algorithms imitate evolution by combining good solutions.