Foundations of Artificial Intelligence

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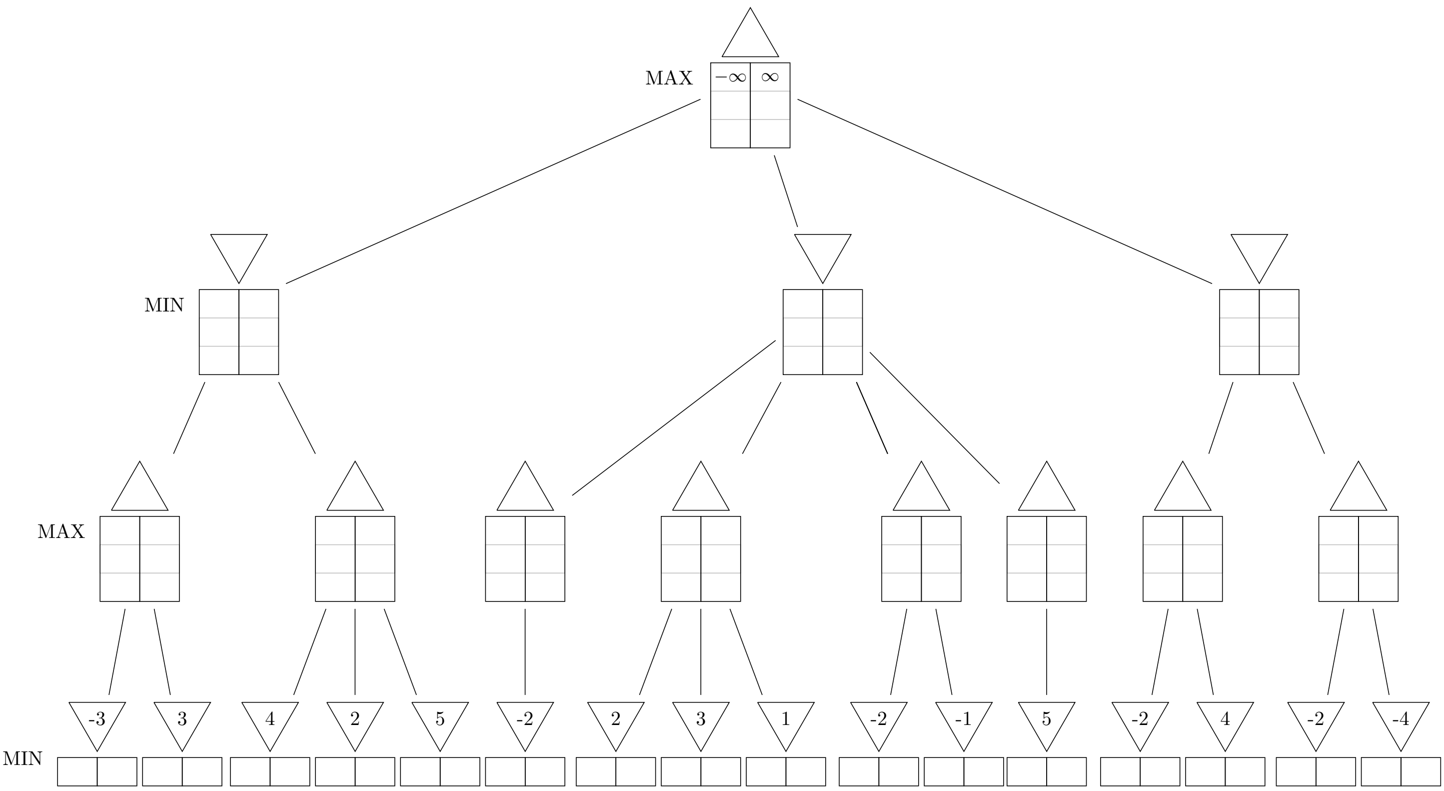
Exercise Sheet 3

Due: Wednesday, May 31, 2017, before the lecture

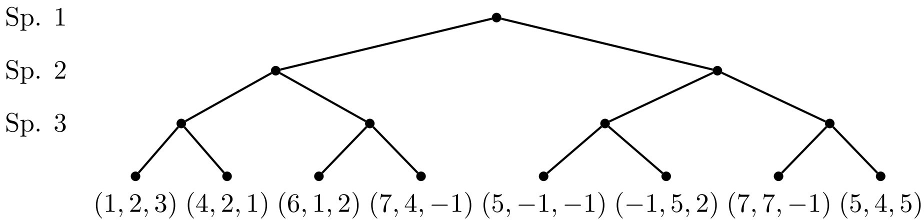
Exercise 3.1 (Board Games)

(a) Consider the game tree for the two-person board game depicted below. Simulate the behavior of the Minimax algorithm with α-β pruning (always expand children from left to right). Enter the computed node values into the triangles and all intermediate α-β values into the appropriate tables.



(b) Consider the problem of search in a three-player game (you may assume that no alliances are allowed) without the zero-sum condition. The players are called 1, 2, and 3. Unlike in the case of two-player zero-sum games, the evaluation function now returns a triple (x_1, x_2, x_3) such that x_i is the value the node has for player i.

Complete the game tree given below by annotating all interior nodes and the root node with the backed-up value triples.



(c) Assume that the value triple $(5, 4, 5)$ at the rightmost leaf node is replaced by $(5, 4, -1)$. Which problem arises now when you try to back up value triples? Suggest how to modify the back-up procedure to obtain a "robust" result at the root node.

Exercise 3.2 (Satisfiability, Models)

(a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.

(1) $\mathit{Smoke} \Rightarrow \mathit{Smoke}$

(2) $\mathit{Smoke} \Rightarrow \mathit{Fire}$

(3) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow (\neg \mathit{Fire} \Rightarrow \neg \mathit{Smoke})$

(4) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow ((\mathit{Smoke} \wedge \mathit{Heat}) \Rightarrow \mathit{Fire})$

(5) $\mathit{Spring} \Leftrightarrow \mathit{SunnyWeather}$

(b) Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following formulae? Explain.

(1) $(A \wedge B) \vee (B \wedge C)$

(2) $A \vee B$

(3) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 3.3 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ, and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \tag{1}$$
$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \tag{2}$$
$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \tag{3}$$
$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \tag{4}$$
$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \tag{5}$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.

Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show whether $K \models (\neg B \rightarrow (A \wedge C))$ holds.

Exercise 3.4 (DPLL)

Use the Davis-Putnam-Logemann-Loveland (DPLL) procedure to find a satisfying assignment for the formula ϕ. Write down all steps carried out by the algorithm during the process. If you have to apply a splitting rule, split on variables in alphabetical order, trying *true* first, then *false*. Indicate the satisfying assignment.

$\phi = (\neg A \vee C \vee \neg D) \wedge (A \vee B \vee C \vee \neg D) \wedge (\neg A \vee \neg E) \wedge \neg C \wedge (A \vee D) \wedge (A \vee C \vee E) \wedge (D \vee E)$

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.