Foundations of Artificial Intelligence

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Exercise Sheet 4

Due: Wednesday, June 21, 2017, before the lecture

Exercise 4.1 (Satisfiability)

Decide whether the following PL1 formulas are satisfiable, unsatisfiable or valid. If a formula is satisfiable, but not valid, construct one interpretation I which is a model, and another one which is not a model of that formula.

The predicates have hereby the following meaning:

man(x)	x is a man
barber(x)	x is a barber
shaves(x,y)	x shaves y
InBar(x)	x is in a bar
drinks(x)	x is drinking

(a) $\exists x\,(man(x) \wedge \forall y\,[man(y) \Rightarrow (shaves(x,y) \iff \neg shaves(y,y))])$

(b) $\exists x\,(barber(x) \wedge \forall y\,[man(y) \Rightarrow (shaves(x,y) \iff \neg shaves(y,y))])$

(c) $\exists x\,[(InBar(x) \wedge drinks(x)) \Rightarrow \forall y(InBar(y) \Rightarrow drinks(y))]$

Exercise 4.2 (Clausal Normal Form)

(a) Transform the following PL1 formulas in Clausal Normal Form.

1. $\forall x\big[\forall z Q(x,z) \Rightarrow \neg\exists y\,(R(y) \wedge P(x,y))\,\big] \wedge \exists y \neg R(y)$
2. $\exists x\big(\forall y Q(x,y) \iff \exists z\big[Q(x,z) \vee P(z)\big]\big)$
3. $\forall x \neg\forall y\big[R(x,y) \Rightarrow \big(\neg P(x) \wedge \exists z Q(x,z,y) \wedge R(y,y)\big)\big]$

(b) Let ϕ be a PL1-formula with

$$\phi = \phi_1 \circ \phi_2,$$

where ϕ_1 and ϕ_2 are PL1-formulas in Prenex Normal Form, and $\circ$ is either $\wedge$ or $\vee$. As far as the satisfiability of ϕ is concerned, does it make a difference whether ϕ_1 and ϕ_2 are skolemized first, or ϕ is skolemized after it was brought into Prenex Normal Form? What consequences does this have for the production of the Clausal Normal Form?

Exercise 4.3 (Unification)

Find (if possible) the most general unifier with the algorithm presented in the lecture.

(a) $\{P(x, f(x), y), P(z, y, x)\}$

(b) $\{Q(x, g(y, x)), Q(\tilde{y}, z), Q(g(\tilde{z}, \tilde{x}), z)\}$

(c) $\{R(y, f(x, y), g(z)), R(g(x), \tilde{z}, y)\}$

Exercise 4.4 (Resolution)

Consider the following statements about the set of natural numbers:

i If x is divisible by y, then x is greater than or equal to y.

ii If x is greater than or equal to y and y is greater than or equal to x than x is equal to y.

iii If x is divisible by y and y is divisible by x then x is equal to y.

(a) Formalize the statements (i)-(iii) in PL1 using appropriate predicates.

(b) Use resolution to show, $(i) \wedge (ii) \models (iii)$ holds or not.

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.