Exercise 5.1 (Conditional Independence)
Suppose you are given a bag containing $n$ unbiased coins, out of which $n - 1$ are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

(a) Suppose you reach into the bag, pick out a coin uniformly at random, toss it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?

(b) Suppose you continue flipping the coin for a total of $k$ times after picking it and see $k$ heads. Now what is the conditional probability that you picked the fake coin?

(c) Suppose you wanted to decide whether the chosen coin was fake by flipping it $k$ times. The decision procedure returns FAKE if all $k$ flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error?

Exercise 5.2 (Bayes Rules)
In Freiburg 80% of all cars are red. You see a car at night that does not appear red to you. You know that you can correctly identify a red car only in 70% of all cases, given the car is red. However you identify a non-red car correctly in 90% of the cases.

(a) List all conditional and non-conditional probabilities that you can determine directly from the task description. Note: Differentiate between the statement that a car is red and the statement that you have seen a red car.

(b) Compute the probability that the car is actually red, wenn you perceive a car as red in Freiburg at night.
Exercise 5.3 (Bayesian Networks)

Consider the following Bayesian network:

(a) Determine which of the following conditional independence statements follow from the structure of the Bayesian network ($\text{Ind}(U, V | W)$ denotes that $U$ is conditionally independent of $V$ given $W$, and $\text{Ind}(U, V)$ denotes unconditional independence of $U$ and $V$):

(i) $\text{Ind}(\text{Cold}, \text{Winter})$
(ii) $\text{Ind}(\text{Winter}, \text{NegligentDriver})$
(iii) $\text{Ind}(\text{Winter}, \text{RadioSilent} | \text{BatteryProblem})$
(iv) $\text{Ind}(\text{Winter}, \text{EngineNotStarting} | \text{BatteryProblem})$
(v) $\text{Ind}(\text{Cold}, \text{NegligentDriver} | \text{RadioSilent})$

(b) Compute $P(\text{EngineNotStarting} | \text{NegligentDriver}, \neg \text{Cold})$. The relevant entries in the conditional probability tables are given below:

- $P(\text{LightsOnOverNight} | \text{NegligentDriver}) = 0.3$
- $P(\text{LightsOnOverNight} | \neg \text{NegligentDriver}) = 0.02$
- $P(\text{TankEmpty} | \text{NegligentDriver}) = 0.1$
- $P(\text{TankEmpty} | \neg \text{NegligentDriver}) = 0.01$
- $P(\text{BatteryProblem} | \text{Cold}, \text{LightsOnOverNight}) = 0.9$
- $P(\text{BatteryProblem} | \neg \text{Cold}, \neg \text{LightsOnOverNight}) = 0.2$
- $P(\text{BatteryProblem} | \neg \text{Cold}, \text{LightsOnOverNight}) = 0.8$
- $P(\text{BatteryProblem} | \text{Cold}, \neg \text{LightsOnOverNight}) = 0.01$
- $P(\text{EngineNotStarting} | \text{BatteryProblem}, \text{TankEmpty}) = 0.9$
- $P(\text{EngineNotStarting} | \neg \text{BatteryProblem}, \neg \text{TankEmpty}) = 0.7$
- $P(\text{EngineNotStarting} | \text{BatteryProblem}, \text{TankEmpty}) = 0.8$
- $P(\text{EngineNotStarting} | \neg \text{BatteryProblem}, \text{TankEmpty}) = 0.05$

(c) List all nodes in the Markov blanket for node $\text{LightsOnOverNight}$. 

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.