Board games are one of the oldest branches of AI (Shannon and Turing 1950).

- Board games present a very abstract and pure form of competition between two opponents and clearly require a form of “intelligence”.
- The states of a game are easy to represent.
- The possible actions of the players are well-defined.

→ Realization of the game as a search problem
→ The individual states are fully accessible
→ It is nonetheless a contingency problem, because the characteristics of the opponent are not known in advance.
Board games are not only difficult because they are contingency problems, but also because the search trees can become astronomically large.

**Examples:**

- **Chess:** On average $35$ possible actions from every position; often, games have $50$ moves per player, resulting in a search depth of $100$: $35^{100} \approx 10^{150}$ nodes in the search tree (with “only” $10^{40}$ legal chess positions).

- **Go:** On average $200$ possible actions with ca. $300$ moves $200^{300} \approx 10^{700}$ nodes.

**Good game programs have the properties that they**

- delete irrelevant branches of the game tree,
- use good evaluation functions for in-between states, and
- look ahead as many moves as possible.
Players are \textbf{MAX} and \textbf{MIN}, where \textbf{MAX} begins.

Initial position (e.g., board arrangement)

Operators (\(=\) legal moves)

Termination test, determines when the game is over. Terminal state = game over.

Strategy. In contrast to regular searches, where a path from beginning to end is simply a solution, \textbf{MAX} must come up with a strategy to reach a terminal state \textit{regardless of what} \textbf{MIN} \textit{does} \(\rightarrow\) correcting reactions to all of \textbf{MIN}’s moves.
Every step of the search tree, also called game tree, is given the player’s name whose turn it is (MAX- and MIN-steps).

When it is possible, as it is here, to produce the full search tree (game tree), the minimax algorithm delivers an optimal strategy for MAX.
1. Generate the complete game tree using depth-first search.

2. Apply the utility function to each terminal state.

3. Beginning with the terminal states, determine the utility of the predecessor nodes as follows:
   - Node is a \texttt{MIN}-node
     Value is the \texttt{minimum} of the successor nodes
   - Node is a \texttt{MAX}-node
     Value is the \texttt{maximum} of the successor nodes
   - From the initial state (root of the game tree), \texttt{MAX} chooses the move that leads to the highest value (\textit{minimax decision}).

\textbf{Note:} Minimax assumes that \texttt{MIN} plays perfectly. Every weakness (i.e., every mistake \texttt{MIN} makes) can only improve the result for \texttt{MAX}.
Minimax Example

MAX

MIN

A_1

A_2

A_3

A_{11} A_{12} A_{13}

A_{21} A_{22} A_{23}

A_{31} A_{32} A_{33}

3 12 8 2 4 6 14 5 2
Minimax Algorithm

Recursively calculates the best move from the initial state.

```plaintext
function MINIMAX-DECISION(state) returns an action
    return arg max_a ∈ ACTIONS(state) MIN-VALUE(Result(state, a))

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← -∞
    for each a in ACTIONS(state) do
        v ← MAX(v, MIN-VALUE(Result(s, a)))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for each a in ACTIONS(state) do
        v ← MIN(v, MAX-VALUE(Result(s, a)))
    return v
```

Note: Minimax can only be applied to game trees that are not too deep. Otherwise, the minimax value must be approximated at a certain level.
Evaluation Function

When the search tree is too large, it can be expanded to a certain depth only. The art is to correctly evaluate the playing position of the leaves of the tree at that depth.

Example of simple evaluation criteria in chess:

- Material value: pawn 1, knight/bishop 3, rook 5, queen 9
- Other: king safety, good pawn structure
- Rule of thumb: three-point advantage = certain victory

The choice of the evaluation function is decisive! The value assigned to a state of play should reflect the chances of winning, i.e., the chance of winning with a one-point advantage should be less than with a three-point advantage.
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The value assigned to a state of play should reflect the chances of winning, i.e., the chance of winning with a one-point advantage should be less than with a three-point advantage.
The preferred evaluation functions are weighted, linear functions:

\[ w_1 f_1 + w_2 f_2 + \cdots + w_n f_n \]

where the \( w \)'s are the weights, and the \( f \)'s are the features. [e.g., \( w_1 = 3 \), \( f_1 = \) number of our own knights on the board]

The above linear sum makes the strong assumption that the contributions of all features are independent. (not true: e.g., bishops in the endgame are more powerful, when there is more space)

The weights can be learned. The features, however, are often designed by human intuition and understanding.
Motivation: Return an answer within the allocated time.

- Fixed-depth search.
- Better: iterative deepening search (stop, when time is over).
- but only stop and evaluate at “quiescent” positions that will not cause large fluctuations in the evaluation function in the following moves. For example, if one can capture a figure, then the position is not “quiescent” because this action might change the evaluation substantially. An alternative is to continue the search at non quiescent positions, preferably by only allowing certain types of moves (e.g., capturing) to reduce search effort, until a quiescent position was reached.
- There still is the problem of limited depth search: horizon effect (see next slide).
Black to move

- Black has a slight material advantage
- ... but will eventually lose (pawn becomes a queen).
- A fixed-depth search cannot detect this because it thinks it can avoid it (on the other side of the horizon—because black is concentrating on the check with the rook, to which white must react).
Can we improve this?
Can we improve this? We do not need to consider all nodes.
If $m > n$ we will never reach node $n$ in the game.
Alpha-Beta Pruning

Minimax algorithm with depth-first search

\( \alpha = \) the value of the best (i.e., highest-value) choice we have found so far at any choice point along the path for MAX.

\( \beta = \) the value of the best (i.e., lowest-value) choice we have found so far at any choice point along the path for MIN.
When Can we Prune?

The following applies:

\( \alpha \) values of MAX nodes can never decrease.

\( \beta \) values of MIN nodes can never increase.

(1) Prune below the MIN node whose \( \beta \)-bound is less than or equal to the \( \alpha \)-bound of its MAX-predecessor node.

(2) Prune below the MAX node whose \( \alpha \)-bound is greater than or equal to the \( \beta \)-bound of its MIN-predecessor node.

→ Provides the same results as the complete minimax search to the same depth (because only irrelevant nodes are eliminated).
function \texttt{ALPHA-BETA-SEARCH}(state) returns an action
\[ v \leftarrow \text{MAX-VALUE}(state, -\infty, +\infty) \]
\textbf{return} the action in ACTIONS(state) with value \( v \)

function \texttt{MAX-VALUE}(state, \( \alpha, \beta \)) returns a utility value
\[ v \leftarrow -\infty \]
\textbf{if} TERMINAL-TEST(state) \textbf{then return} UTILITY(state)
\textbf{for each} \( a \) \textbf{in} ACTIONS(state) \textbf{do}
\[ v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{RESULT}(s,a), \alpha, \beta)) \]
\textbf{if} \( v \geq \beta \) \textbf{then return} \( v \)
\[ \alpha \leftarrow \text{MAX}(\alpha, v) \]
\textbf{return} \( v \)

function \texttt{MIN-VALUE}(state, \( \alpha, \beta \)) returns a utility value
\[ v \leftarrow +\infty \]
\textbf{if} TERMINAL-TEST(state) \textbf{then return} UTILITY(state)
\textbf{for each} \( a \) \textbf{in} ACTIONS(state) \textbf{do}
\[ v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(\text{RESULT}(s,a) , \alpha, \beta)) \]
\textbf{if} \( v \leq \alpha \) \textbf{then return} \( v \)
\[ \beta \leftarrow \text{MIN}(\beta, v) \]
\textbf{return} \( v \)

Initial call with \texttt{MAX-VALUE}(\texttt{initial-state}, -\infty, +\infty)
Alpha-Beta Pruning Example

MAX

MIN

3
12
8

≥ 3
Alpha-Beta Pruning Example

MAX

MIN

3
12
8
2

X
X

≥ 3

≥ 2

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Alpha-Beta Pruning Example

MAX

MIN

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Alpha-Beta Pruning Example

MAX

MIN

3 12 8
3 2
2
X X
14
14
5
5
3

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Alpha-Beta Pruning Example

MAX

MIN

3
12
8
2
X
14
5
2
3

3
3
2
2
X
X
14
5
2
2

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Efficiency Gain

- The alpha-beta search cuts the largest amount off the tree when we examine the best move first.

- In the best case (always the best move first), the search expenditure is reduced to $O\left(\frac{b^d}{2}\right) \Rightarrow$ we can search twice as deep in the same amount of time.

- In the average case (randomly distributed moves), for moderate $b$ ($b < 100$), we roughly have $O\left(\frac{b^{3d}}{4}\right)$.

- However, the best move typically is not known. Practical case: A simple ordering heuristic brings the performance close to the best case $\Rightarrow$ In chess, we can thus reach a depth of 6–7 moves.

Good ordering for chess?
Efficiency Gain

- The alpha-beta search **cuts the largest amount off the tree** when we examine the **best move first**.

- In the **best case** (always the best move first), the search expenditure is reduced to $O(b^{d/2}) \Rightarrow$ we can search twice as deep in the same amount of time.

- In the average case (randomly distributed moves), for moderate $b$ ($b < 100$), we roughly have $O(b^{3d/4})$.

- However, the best move typically is not known. **Practical case**: A simple ordering heuristic brings the performance close to the best case $\Rightarrow$ In chess, we can thus reach a depth of 6–7 moves.

**Good ordering for chess?** Try captures first, then threats, then forward moves, then backward moves.
White has just rolled a 6 and a 5 and has 4 legal moves.
In addition to MIN- and MAX nodes, we need chance nodes (for the dice).
Utility function for chance nodes $C$ over MAX:

- $d_i$: possible dice roll
- $P(d_i)$: probability of obtaining that roll
- $S(C, d_i)$: attainable positions from $C$ with roll $d_i$
- $\text{UTILITY}(s)$: Evaluation of $s$

$$\text{EXPECTIMAX}(C) = \sum_i P(d_i) \max_{s \in S(C,d_i)} (\text{UTILITY}(s))$$

$\text{EXPECTIMIN}$ likewise
Order-preserving transformations on the evaluation values may change the best move:

Search costs increase: Instead of $O(b^d)$, we get $O((b \times n)^d)$, where $n$ is the number of possible dice outcomes.

→ In Backgammon ($n = 21$, $b = 20$, can be 4000) the maximum for $d$ is 2.
Recently card games such as bridge and poker have been addressed as well.

One approach: simulate play with open cards and then average over all possible plays (or make a Monte Carlo simulation) using minimax (perhaps modified).

Pick the move with the best expected result (usually all moves will lead to a loss, but some give better results).

- Averaging over clairvoyance

Although “incorrect”, appears to give reasonable results.
State of the Art (1)

**Backgammon**: The *BKG* program defeated the official world champion in 1980. A newer program TD-Gammon is among the top 3 players.

**Checkers, draughts** (by international rules): A program called *Chinook* is the official world champion in man-computer competition (acknowledges by ACF and EDA) and is the highest-rated player:

- Chinook: 2712
- Ron King: 2632
- Asa Long: 2631
- Don Lafferty: 2625

In 1995, Chinook won a 32 game match against Don Lafferty.

**Othello**: Very good, even on normal computers. In 1997, the *Logistello* program defeated the human world champion.

**Chess**: In 1997, world chess master G. Kasparow was beaten by a computer in a match of 6 games by *Deep Blue* (IBM Thomas J. Watson Research Center).

Special hardware (32 processors with 8 chips, 2 Mi. calculations per second) and special chess knowledge.
Go: The program *AlphaGo* was able to beat in March 2016 one of the best human players Lee Sedol (according to ELO ranking the 4th best player worldwide) 4:1. AlphaGo used Monte Carlo search techniques (UCT) and deep learning techniques.

Poker: In January 2017, *Libratus* played against four top-class human poker players for 20 days *heads-up no-limit Texas hold ’em*. In the end, Libratus was more than 1.7 M$ ahead. Libratus used a number of different techniques all based on game theory.
The Reasons for Success...

- Alpha-Beta-Search
- ... with dynamic decision-making for uncertain positions
- Good (but usually simple) evaluation functions
- Large databases of opening moves
- Very large game termination databases (for checkers, all ten-piece situations)
- For Go, Monte-Carlo and machine learning techniques proved to be successful.
- ... and very fast and parallel processors, huge memory, and plenty of plays.
- For Poker, game theoretic analysis together with extensive self-play (15 million core CPU hours) were important.
A game can be defined by the initial state, the operators (legal moves), a terminal test and a utility function (outcome of the game).

In two-player board games, the minimax algorithm can determine the best move by enumerating the entire game tree.

The alpha-beta algorithm produces the same result but is more efficient because it prunes away irrelevant branches.

Usually, it is not feasible to construct the complete game tree, so the utility of some states must be determined by an evaluation function.

Games of chance can be handled by an extension of the alpha-beta algorithm.

The success for different games is based on quite different methodologies.