Introduction to Mobile Robotics

Wheeled Locomotion

Wolfram Burgard
Locomotion of Wheeled Robots

Locomotion (Oxford Dict.): Power of motion from place to place

- Differential drive (AmigoBot, Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive (B21)
- XR4000
- Mecanum wheels

we also allow wheels to rotate around the z axis
Instantaneous Center of Curvature

For rolling motion to occur, each wheel has to move along its y-axis
Differential Drive

\[ ICC = [x - R \sin \theta, y + R \cos \theta] \]

\[ \omega(R + l/2) = v_r \]
\[ \omega(R - l/2) = v_l \]
\[ R = \frac{l}{2} \left( \frac{v_l + v_r}{v_r - v_l} \right) \]
\[ \omega = \frac{v_r - v_l}{l} \]
\[ v = \frac{v_r + v_l}{2} \]
### Differential Drive: Forward Kinematics

The forward kinematics of a differential drive robot can be described by the following equations:

- The position vector $\mathbf{p}(t) = [x(t), y(t), \theta(t)]^T$ at time $t$ is given by:
  
  $\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega\delta t \end{bmatrix}$

- The change in position from time $t$ to $t + \delta t$ is:
  
  $\begin{bmatrix} x(t + \delta t) - x(t) \\ y(t + \delta t) - y(t) \\ \theta(t + \delta t) - \theta(t) \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta x(t) \\ \delta y(t) \\ \delta \theta(t) \end{bmatrix}$

- The position $x(t)$ at time $t$ is:
  
  $x(t) = \int_{0}^{t} v(t') \cos[\theta(t')] dt'$

- The position $y(t)$ at time $t$ is:
  
  $y(t) = \int_{0}^{t} v(t') \sin[\theta(t')] dt'$

- The orientation $\theta(t)$ at time $t$ is:
  
  $\theta(t) = \int_{0}^{t} \omega(t') dt'$
Differential Drive: Forward Kinematics

\[
\begin{bmatrix}
    x' \\
    y' \\
    \theta'
\end{bmatrix}
= \begin{bmatrix}
    \cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\
    \sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x - ICC_x \\
    y - ICC_y \\
    \theta
\end{bmatrix}
+ \begin{bmatrix}
    ICC_x \\
    ICC_y \\
    \omega \delta t
\end{bmatrix}
\]

\[
x(t) = \frac{1}{2} \int_{0}^{t} [v_r(t') + v_l(t')] \cos[\theta(t')] dt'
\]

\[
y(t) = \frac{1}{2} \int_{0}^{t} [v_r(t') + v_l(t')] \sin[\theta(t')] dt'
\]

\[
\theta(t) = \frac{1}{l} \int_{0}^{t} [v_r(t') - v_l(t')] dt'
\]
Ackermann Drive

\[ ICC = [x - R \sin \theta, y + R \cos \theta] \]

\[ R = \frac{d}{\tan \varphi} \]

\[ \omega(R + l/2) = v_r \]
\[ \omega(R - l/2) = v_l \]

\[ R = \frac{l}{2} \frac{(v_l + v_r)}{(v_r - v_l)} \]

\[ \omega = \frac{v_r - v_l}{l} \]
Synchronous Drive

\[
x(t) = \int_0^t v(t') \cos[\theta(t')] \, dt'
\]

\[
y(t) = \int_0^t v(t') \sin[\theta(t')] \, dt'
\]

\[
\theta(t) = \int_0^t \omega(t') \, dt'
\]
XR4000 Drive

\[
x(t) = \int_{0}^{t} v(t') \cos[\theta(t')] dt'
\]

\[
y(t) = \int_{0}^{t} v(t') \sin[\theta(t')] dt'
\]

\[
\theta(t) = \int_{0}^{t} \omega(t') dt'
\]
Mecanum Wheels

\[ v_y = \frac{(v_0 + v_1 + v_2 + v_3)}{4} \]

\[ v_x = \frac{(v_0 - v_1 + v_2 - v_3)}{4} \]

\[ v_\theta = \frac{(v_0 + v_1 - v_2 - v_3)}{4} \]

\[ v_{error} = \frac{(v_0 - v_1 - v_2 + v_3)}{4} \]
The Kuka OmniRob Platform
Example: KUKA youBot
Tracked Vehicles
Other Robots: OmniTread

[courtesy by Johann Borenstein]
Non-Holonomic Constraints

- Non-holonomic constraints limit the possible incremental movements within the configuration space of the robot.
- Robots with differential drive or synchro-drive move on a circular trajectory and cannot move sideways.
- Mecanum-wheeled robots can move sideways (they have no non-holonomic constraints).
Holonomic vs. Non-Holonomic

- Non-holonomic constraints reduce the control space with respect to the current configuration
  - E.g., moving sideways is impossible.

- Holonomic constraints reduce the configuration space.
  - E.g., a train on tracks (not all positions and orientations are possible)
Drives with Non-Holonomic Constraints

- Synchro-drive
- Differential drive
- Ackermann drive
Drives without Non-Holonomic Constraints

- Mecanum wheels
Dead Reckoning and Odometry

- Estimating the motion based on the issued controls/wheel encoder readings
- Integrated over time
Summary

- Introduced different types of drives for wheeled robots
- Math to describe the motion of the basic drives given the speed of the wheels
- Non-holonomic constraints
- Odometry and dead reckoning