Introduction to Mobile Robotics

Information Driven Exploration

Wolfram Burgard
Tasks of Mobile Robots

- mapping
- localization
- path planning
- exploration
- active localization
- integrated approaches

SLAM
Exploration and SLAM

- SLAM is typically **passive**, because it consumes incoming sensor data
- Exploration **actively guides the robot** to cover the environment with its sensors
- Exploration in combination with SLAM: **Acting under pose and map uncertainty**
- Uncertainty should/needs to be taken into account when selecting an action
Mapping with Rao-Blackwellized Particle Filter (Brief Summary)

- Each particle represents a possible trajectory of the robot
- Each particle
  - maintains its own map and
  - updates it upon “mapping with known poses”
- Each particle survives with a probability proportional to the likelihood of the observations relative to its own map
Factorization Underlying Rao-Blackwellized Mapping

\[ p(x, m \mid z, u) \]

\[ = p(m \mid x, z, u) p(x \mid z, u) \]

Mapping with known poses

Particle filter representing trajectory hypotheses
Example: Particle Filter for Mapping

3 particles

map of particle 1

map of particle 2

map of particle 3
Combining Exploration and SLAM
SLAM approaches seen so far are purely passive

By reasoning about control, the mapping process can be made much more effective

Question: Where to move next?
Where to Move Next?
Decision-Theoretic Approach

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

Utility = uncertainty reduction - cost
Example

high pose uncertainty
The Uncertainty of a Posterior

- **Entropy** is a general measure for the uncertainty of a posterior

\[
H(X) = - \int p(X = x) \log p(X = x) \, dx \\
= \mathbb{E}_X[- \log(p(X))] 
\]

- **Conditional Entropy**

\[
H(X \mid Y) = \int p(Y = y) H(X \mid Y = y) \, dy 
\]


Mutual Information

- **Expected Information Gain or Mutual Information** = Expected Uncertainty Reduction

\[
I(X;Y) = H(X) - H(X \mid Y)
\]

\[
I(X;Y) = H(Y) - H(Y \mid X)
\]

\[
I(X;Y \mid z = c_k) = H(X \mid z = c_k) - H(X \mid Y, z = c_k)
\]

\[
I(X;Y \mid Z) = H(X \mid Z) - H(X \mid Y, Z)
\]
Entropy Computation

\[ H(X, Y) \]

\[ = E_{X,Y} \left[ - \log p(X, Y) \right] \]

\[ = E_{X,Y} \left[ - \log (p(X) \cdot p(Y | X)) \right] \]

\[ = E_{X,Y} \left[ - \log p(X) \right] + E_{X,Y} \left[ - \log p(Y | X) \right] \]

\[ = H(X) + \int_{x,y} -p(x, y) \log p(y | x) \, dx \, dx \]

\[ = H(X) + \int_{x,y} -p(y | x)p(x) \log p(y | x) \, dx \, dy \]

\[ = H(X) + \int_{x} p(x) \int_{y} -p(y | x) \log p(y | y) \, dy \, dx \]

\[ = H(X) + \int_{x} p(x)H(Y | X = x) \, dx \]

\[ = H(X) + H(Y | X) \]
The Uncertainty of the Robot

- The uncertainty of the RBPF:

\[ H(X, M) = H(X) + \int_x p(x) H(M \mid X = x) \, dx \]

\[ H(X, M) = H(X) + \sum_{i=1}^{\text{#particles}} \omega_i H(M^i \mid X^i = x^i) \]

- trajectory uncertainty
- particle weights
- map uncertainty
Computing the Entropy of the Map Posterior

Occupancy Grid map $m$:

$$H(M) = - \sum_{c \in M} p(c) \log p(c) + (1 - p(c)) \log(1 - p(c))$$

- map uncertainty
- grid cells
- probability that the cell is occupied
Map Entropy

The overall entropy is the sum of the individual entropy values.
Computing the Entropy of the Trajectory Posterior

1. High-dimensional Gaussian

\[ H(\mathcal{G}(\mu, \Sigma)) = \log((2\pi e)^{n/2}|\Sigma|) \]

reduced rank for sparse particle sets

2. Grid-based approximation

\[ H(X) \sim \text{const.} \]

for sparse particle clouds
Approximation of the Trajectory Posterior Entropy

Average pose entropy over time:

\[ H(X_{1:t} \mid d) \approx \frac{1}{t} \sum_{t'=1}^{t} H(X_{t'} \mid d) \]
The mutual information $I$ is given by the expected reduction of entropy in the belief action to be carried out

$$I(X, M; Z^a) = \text{“uncertainty of the filter”} - \text{“uncertainty of the filter after carrying out action } a\text{”}$$
Integrating Over Observations

- Computing the mutual information requires to integrate over potential observations

\[
I(X, M; Z^a) = H(X, M) - H(X, M \mid Z^a)
\]

\[
H(X, M \mid Z^a) = \int_z p(z \mid a) H(X, M \mid Z^a = z) \, dz
\]
Approximating the Integral

- The particle filter represents a posterior about possible maps

map of particle 1  map of particle 2  map of particle 3
Approximating the Integral

- The particle filter represents a posterior about possible maps
- Simulate laser measurements in the maps of the particles

\[
H(X, M \mid Z^a) = \sum_{z} p(z \mid a) H(X, M \mid Z^a = z)
\]

\[
= \sum_{i} \omega^{[i]} H(X, M \mid Z^a = z^{[i]}_{sim_a})
\]

measurement sequences simulated in the maps

likelihood (particle weight)
Simulating Observations

- Ray-casting in the map of each particle to generate observation sequences
The Utility

- We take into account the cost of an action: mutual information $\rightarrow$ utility $U$

- Select the action with the highest utility

$$a^* = \text{argmax}_a I(X; M; Z^a) - \text{cost}(a)$$
Focusing on Specific Actions

To efficiently sample actions we consider

- exploratory actions (1-3)
- loop closing actions (4) and
- place revisiting actions (5)
Dual Representation for Loop Detection

- **Trajectory graph** ("topological map") stores the path traversed by the robot
- **Occupancy grid** map represents the space covered by the sensors
- **Loops** correspond to long paths in the trajectory graph and short paths in the grid map
Example: Trajectory Graph
Application Example

high pose uncertainty
Example: Possible Targets
Example: Evaluate Targets

timestep 35

robot
Example: Move Robot to Target
Example: Evaluate Targets

robot

decision at timestep 70

expected utility

target location

0 1 2 3 4 5 6
Example: Move Robot

![Diagram showing decision at timestep 70 with expected utility for different target locations. A path is shown from start to the robot's position.]
Example: Entropy Evolution
Comparison

Map uncertainty only:

After loop closing action:
Real Exploration Example

Selected target location
The decision-theoretic approach leads to **intuitive behaviors**: “re-localize before getting lost”

Some animals show a similar behavior (dogs marooned in the tundra of north Russia)
Summary

- A decision-theoretic approach to exploration in the context of RBPF-SLAM

- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain

- Reasons about measurements obtained along the path of the robot

- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions