Foundations of Artificial Intelligence

Prof. Dr. J. Boedecker, Prof. Dr. W. Burgard, Prof. Dr. F. Hutter, Prof. Dr. B. Nebel
M. Krawez, T. Schulte
Summer Term 2018

University of Freiburg
Department of Computer Science

Exercise Sheet 3
**Due: Wednesday, May 30, 2018, before 12:00**

Exercise 3.1 (Board Games)

(a) Consider the game tree for the two-person board game depicted below.
Simulate the behavior of the Minimax algorithm with α-β pruning (always
expand children from left to right). Enter the computed node values into
the triangles and all intermediate α-β values into the appropriate tables.
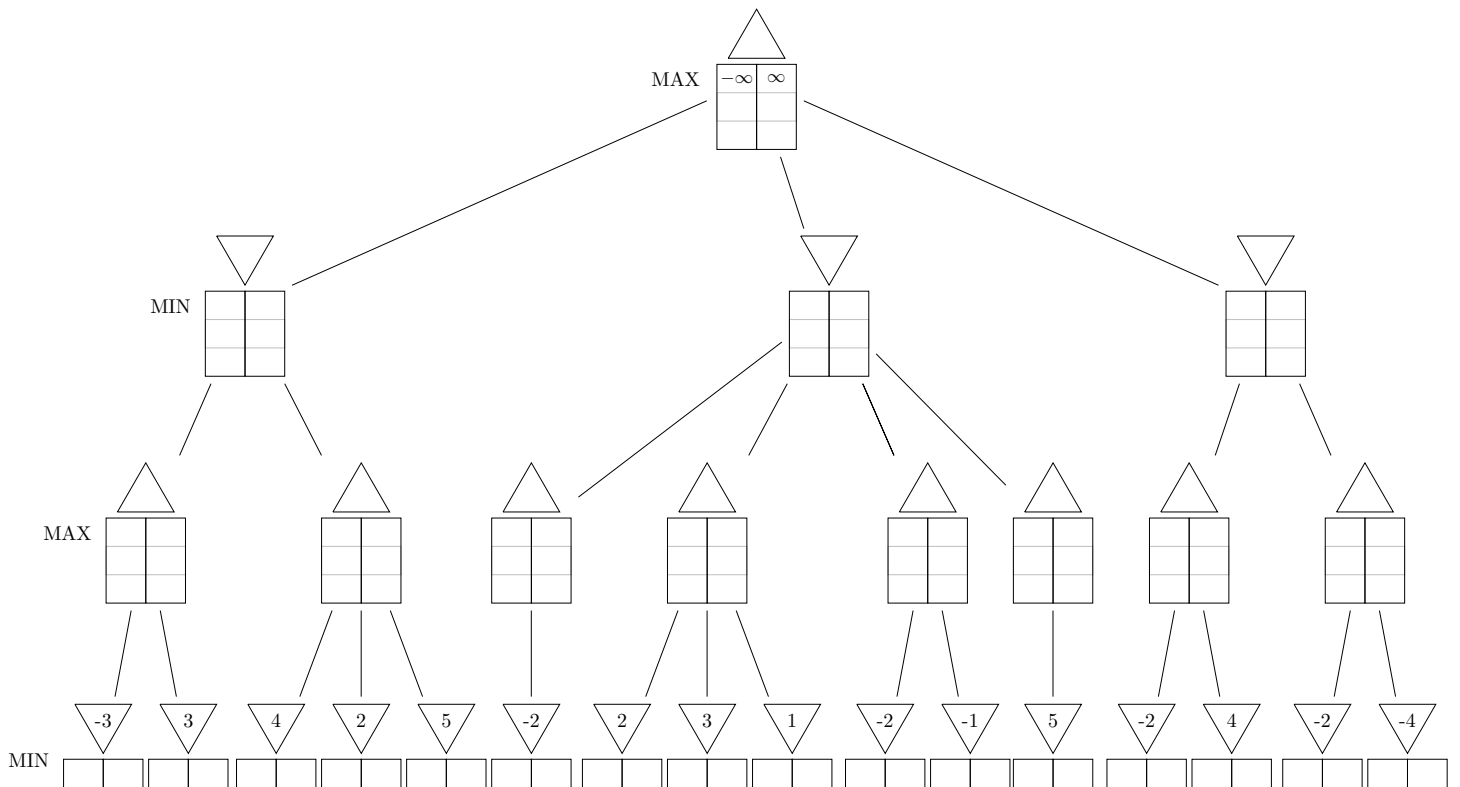
(b) Consider the problem of search in a three-player game (you may assume that no alliances are allowed) without the zero-sum condition. The players are called 1, 2, and 3. Unlike in the case of two-player zero-sum games, the evaluation function now returns a triple (x_1, x_2, x_3) such that x_i is the value the node has for player i.

Complete the game tree given below by annotating all interior nodes and the root node with the backed-up value triples.

Sp. 1

Sp. 2

Sp. 3

$(1, 2, 3)$ $(4, 2, 1)$ $(6, 1, 2)$ $(7, 4, -1)$ $(5, -1, -1)$ $(-1, 5, 2)$ $(7, 7, -1)$ $(5, 4, 5)$
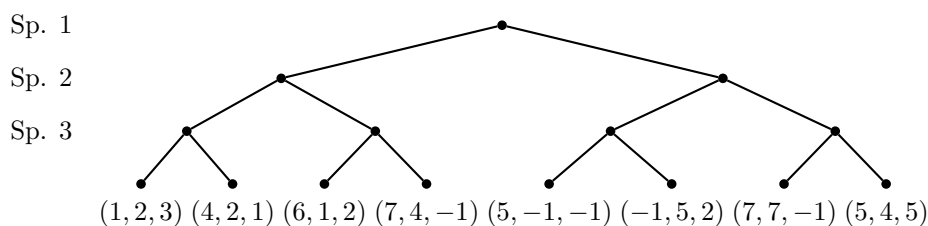
(c) Assume that the value triple $(5, 4, 5)$ at the rightmost leaf node is replaced by $(5, 4, -1)$. Which problem arises now when you try to back up value triples? Suggest how to modify the back-up procedure to obtain a "robust" result at the root node.

Exercise 3.2 (Satisfiability, Models)

(a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.

(1) $Smoke \Rightarrow Smoke$

(2) $Smoke \Rightarrow Fire$

(3) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Fire \Rightarrow \neg Smoke)$

(4) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$

(5) $Spring \Leftrightarrow SunnyWeather$

(b) Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following formulae? Explain.

(1) $(A \wedge B) \vee (B \wedge C)$

(2) $A \vee B$

(3) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 3.3 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ, and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \tag{1}$$
$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \tag{2}$$
$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \tag{3}$$
$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \tag{4}$$
$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \tag{5}$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.

Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show whether $K \models (\neg B \rightarrow (A \wedge C))$ holds.

Exercise 3.4 (Modeling, Proofs)

Consider the following knowledge base:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Using this knowledge base, can you prove that the unicorn is (a) mythical, (b) magical or (c) horned? First, formalize the knowledge base with propositional logic. If a statement is valid or unsatisfiable, use resolution for prove. Else, write down one satisfying and one unsatisfying interpretation.

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names on your solution.