Foundations of Artificial Intelligence

Prof. Dr. J. Boedecker, Prof. Dr. W. Burgard, Prof. Dr. F. Hutter, Prof. Dr. B. Nebel
M. Krawez, T. Schulte
Summer Term 2018

University of Freiburg
Department of Computer Science

Exercise Sheet 4

Due: Wednesday, June 13, 2018, before 12:00

Exercise 4.1 (DPLL)
Use the Davis-Putnam-Logemann-Loveland (DPLL) procedure to find a satisfying assignment for the formula ϕ_i. Write down all steps carried out by the algorithm during the process. If you have to apply a splitting rule, split on variables in alphabetical order, trying *true* first, then *false*. Indicate the satisfying assignment.

(a)

$$\phi_1 = (\neg A \vee C \vee \neg D) \wedge (A \vee B \vee C \vee \neg D) \wedge (\neg A \vee \neg E) \wedge \neg C \wedge (A \vee D) \wedge (A \vee C \vee E) \wedge (D \vee E)$$

(b)

$$\phi_2 = (E \vee A) \wedge (B \vee \neg A \vee C) \wedge (E \vee \neg D) \wedge (B \vee \neg C) \wedge (\neg B \vee D) \wedge (\neg E \vee \neg A \vee \neg D \vee \neg B)$$

Exercise 4.2 (Semantics of Predicate Logic)
Consider the Interpretation $\mathcal{I} = < \mathcal{D}, \cdot^{\mathcal{I}} >$ with

- $D = \{0, 1, 2, 3\}$
- $even^{\mathcal{I}} = \{0, 2\}$
- $odd^{\mathcal{I}} = \{1, 3\}$
- $lessThan^{\mathcal{I}} = \{(0,1), (0,2), (0,3), (1,2), (1,3), (2,3)\}$
- $two^{\mathcal{I}} = 2$
- $plus^{\mathcal{I}} : D \times D \to D, plus^{\mathcal{I}}(a, b) = (a + b) \mod 4$

and the variable assignment $\alpha = \{(x, 0), (y, 1)\}$.
Decide for the following formulae θ_i if $\mathcal{I}$ is a model for θ_i under α, i.e. if $\mathcal{I}, \alpha \models \theta_i$.
Explain your answer by formally applying the semantics.

(a) $\theta_1 = odd(y) \wedge even(two)$

(b) $\theta_2 = \forall x \ (even(x) \vee odd(x))$

(c) $\theta_3 = \forall x \exists y \ lessThan(x, y)$

(d) $\theta_4 = \forall x \ (even(x) \Rightarrow \exists y \ lessThan(x, y))$

(e) $\theta_5 = \forall x \ (odd(x) \Rightarrow even(plus(x, y)))$

The exercise sheets may and should be worked on in groups of three (3) students.
Please write all your names on your solution.