Exercise 4.1 (DPLL)

Use the Davis-Putnam-Logemann-Loveland (DPLL) procedure to find a satisfying assignment for the formula $\phi_i$. Write down all steps carried out by the algorithm during the process. If you have to apply a splitting rule, split on variables in alphabetical order, trying true first, then false. Indicate the satisfying assignment.

(a) $\phi_1 = (\neg A \lor C \lor \neg D) \land (A \lor B \lor C \lor \neg D) \land (\neg A \lor \neg E) \land \neg C \land (A \lor D) \land (A \lor C \lor E) \land (D \lor E)$

(b) $\phi_2 = (E \lor A) \land (B \lor \neg A \lor C) \land (E \lor \neg D) \land (B \lor \neg C) \land (\neg B \lor D) \land (\neg E \lor \neg A \lor D \lor \neg B)$

Exercise 4.2 (Semantics of Predicate Logic)

Consider the Interpretation $I = \langle D, T \rangle$ with

- $D = \{0, 1, 2, 3\}$
- $\text{even}^T = \{0, 2\}$
- $\text{odd}^T = \{1, 3\}$
- $\text{lessThan}^T = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)\}$
- $\text{two}^T = 2$
- $\text{plus}^T : D \times D \to D$, $\text{plus}^T(a, b) = (a + b) \mod 4$

and the variable assignment $\alpha = \{(x, 0), (y, 1)\}$.

Decide for the following formulae $\theta_i$ if $I$ is a model for $\theta_i$ under $\alpha$, i.e. if $I, \alpha \models \theta_i$. Explain your answer by formally applying the semantics.

(a) $\theta_1 = \text{odd}(y) \land \text{even}(\text{two})$

(b) $\theta_2 = \forall x (\text{even}(x) \lor \text{odd}(x))$

(c) $\theta_3 = \forall x \exists y \text{ lessThan}(x, y)$

(d) $\theta_4 = \forall x (\text{even}(x) \implies \exists y \text{ lessThan}(x, y))$

(e) $\theta_5 = \forall x (\text{odd}(x) \implies \text{even}(\text{plus}(x, y)))$

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names on your solution.