Foundations of Artificial Intelligence

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Exercise Sheet 5

Due: Wednesday, June 27, 2018, before 12:00

Exercise 5.1 (Clausal Normal Form)

Transform the following PL1 formulas in Clausal Normal Form.

1. $\forall x \big[\forall z Q(x,z) \Rightarrow \neg \exists y \left(R(y) \wedge P(x,y) \right) \big] \wedge \exists y \neg R(y)$
2. $\exists x \big(\forall y Q(x,y) \iff \exists z \big[Q(x,z) \vee P(z) \big] \big)$
3. $\forall x \neg \forall y \big[R(x,y) \Rightarrow \big(\neg P(x) \wedge \exists z Q(x,z,y) \wedge R(y,y) \big) \big]$

Exercise 5.2 (Unification)

Find (if possible) the most general unifier with the algorithm presented in the lecture.

(a) $\{P(x, f(x), y), P(z, y, x)\}$

(b) $\{Q(x, g(y,x)), Q(\tilde{y}, z), Q(g(\tilde{z}, \tilde{x}), z)\}$

(c) $\{R(y, f(x,y), g(z)), R(g(x), \tilde{z}, y)\}$

Exercise 5.3 (Resolution)

Apply the resolution calculus to show that the following clause set is unsatisfiable. In each step, write down which substitution you use.

Clause set as stated in the original exercise sheet (unsolvable):

$$\big\{ Q(x,y), P\big(g(z,z)\big) \big\} \tag{1}$$

$$\big\{ \neg P\big(g(x',a)\big), Q\big(y', g(z',y')\big) \big\} \tag{2}$$

$$\big\{ \neg P(y''), \neg Q\big(f(x''), y''\big), \neg Q(a, z'') \big\} \tag{3}$$

New (solvable) version:

$$\Big\{Q(x,y), P\big(g(z,z)\big)\Big\} \tag{1}$$
$$\Big\{\neg P\big(g(x',a)\big), Q\big(y', g(z',y')\big)\Big\} \tag{2}$$
$$\Big\{\neg Q\big(f(x''), y''\big), \neg Q(a,z'')\Big\} \tag{3}$$

Exercise 5.4 (Proofs with resolution)
Consider the following statements about the set of natural numbers:

i If x is divisible by y, then x is greater than or equal to y.

ii If x is greater than or equal to y and y is greater than or equal to x than x is equal to y.

iii If x is divisible by y and y is divisible by x then x is equal to y.

(a) Formalize the statements (i)-(iii) in PL1 using appropriate predicates.

(b) Use resolution to show, $(i) \wedge (ii) \models (iii)$ holds or not.

Exercise 5.5 (Planning)
Consider the following STRIPS-Task $\Pi = \langle \mathcal{S}, O, I, G \rangle$:

- $\mathcal{S}$: {X, Y, Z, G}
- $O : \{A, B, C, D, E, F\}$ where

$$\begin{aligned}
A: &\ pre(A) = \{X\}, && \mathit{eff}(A) = \{Y, Z\} \\
B: &\ pre(B) = \{X\}, && \mathit{eff}(B) = \{\neg X, Z\} \\
C: &\ pre(C) = \{\neg Y\}, && \mathit{eff}(C) = \{Z\} \\
D: &\ pre(D) = \{\neg Z\}, && \mathit{eff}(D) = \{Y\} \\
E: &\ pre(E) = \{\neg X, Y\}, && \mathit{eff}(E) = \{\neg Y, G\} \\
F: &\ pre(F) = \{Z\}, && \mathit{eff}(F) = \{\neg Z, G\}
\end{aligned}$$

- I: {X, Y}
- G: {G}

(a) State for each operator from O if it is applicable in I or not. For each applicable operator also give the resulting state after applying that operator in I.

Operator	Applicable?	Resulting State
A		
B		
C		
D		
E		
F		

(b) Give an applicable plan π that leads from I to G.

The exercise sheets may and should be worked on in groups of three (3) students.
Please write all your names on your solution.