Rectification of exercise 5.3

Unfortunately, there was a mistake in the formulation of exercise 5.3, which rendered it unsolvable in its original form, i.e., no empty clause could be derived from the given clause set:

$$\Big\{Q(x,y), P\big(g(z,z)\big)\Big\} \tag{1}$$

$$\Big\{\neg P\big(g(x',a)\big), Q\big(y', g(z',y')\big)\Big\} \tag{2}$$

$$\Big\{\neg P(y''), \neg Q\big(f(x''), y''\big), \neg Q(a,z'')\Big\} \tag{3}$$

Also, there was a mistake in the solution presented in the exercise class, namely step four with the substitution $\left\{\frac{x}{a}, \frac{y}{z''}\right\}$ evaluates to $\top$.

$$(1)+(2) \quad \text{with} \quad s = \left\{\frac{z}{a}, \frac{x'}{a}\right\}: \quad \big\{Q(x,y), Q\big(y', g(z',y')\big)\big\} \tag{4}$$

$$(4) \quad \text{with} \quad s = \left\{\frac{x}{y'}, \frac{y}{g(z',y')}\right\}: \quad \big\{Q\big(y', g(z',y')\big)\big\} \tag{5}$$

$$(1)+(3) \quad \text{with} \quad s = \left\{\frac{y''}{g(z,z)}\right\}: \quad \big\{Q(x,y), \neg Q\big(f(x''), g(z,z)\big), \neg Q(a,z'')\big\} \tag{6}$$

$$(6) \quad \text{with} \quad s = \left\{\frac{x}{a}, \frac{y}{z''}\right\}: \quad \cancel{\big\{\neg Q\big(f(x''), g(z,z)\big)\big\}} \quad \top \tag{7}$$

$$\cancel{(7)+(5) \quad \text{with} \quad s = \left\{\frac{y'}{f(x'')}, \frac{z}{f(x'')}, \frac{z'}{f(x'')}\right\}: \quad \bot} \tag{8}$$

However, a slight modification of the initial clause set (remove $\neg P(y'')$ from the 3rd clause)

$$\Big\{Q(x,y), P\big(g(z,z)\big)\Big\} \tag{1}$$
$$\Big\{\neg P\big(g(x',a)\big), Q\big(y', g(z',y')\big)\Big\} \tag{2}$$
$$\Big\{\neg Q\big(f(x''), y''\big), \neg Q(a,z'')\Big\} \tag{3}$$

makes the exercise solvable:

$$(1)+(2) \quad \text{with} \quad s = \left\{\frac{z}{a}, \frac{x'}{a}\right\}: \quad \big\{Q(x,y), Q\big(y', g(z',y')\big)\big\} \tag{4}$$
$$(4) \quad \text{with} \quad s = \left\{\frac{x}{y'}, \frac{y}{g(z',y')}\right\}: \quad \big\{Q\big(y', g(z',y')\big)\big\} \tag{5}$$
$$(3)+(5) \quad \text{with} \quad s = \left\{\frac{y'}{f(x'')}, \frac{y''}{g(z', f(x''))}\right\}: \quad \{\neg Q(a,z'')\} \tag{6}$$
$$(5)+(6) \quad \text{with} \quad s = \left\{\frac{y'}{a}, \frac{z''}{g(z',a)}\right\}: \quad \bot \tag{7}$$