Exercise 2.1 (Informed search)
A house cleaning robot tries to find the shortest path from S (start) to G (goal). The robot can move between horizontally or vertically connected grid cells, one cell in each step. If a wall (thick black line) lies in between two cells, the robot cannot move between them. Each step incurs a uniform cost of 1. Figure (a) shows the initial state, Figure (b) the heuristic value estimates of each cell.

(a) Perform an $A^*$ search to find the shortest path from S to G. For all generated nodes, write down the respective $g$- and $f$-values in the corresponding grid cell. All other cells should be left blank.

(b) What is the definition of an admissible heuristic? Is the heuristic from Figure (b) admissible?

(c) Let $h^*(n)$ be the actual cost of the optimal path from $n$ to the goal $G$. How many nodes does $A^*$ expand when using the $h^*$ heuristic?

Exercise 2.2 (Local search)
We will now examine hill-climbing in the same setting as above (i.e. a robot navigates on a grid facing walls as obstacles).

(a) Explain how hill-climbing would work as a method of reaching a particular end point.
(b) Show how a combination of walls that form a non-convex obstacle can result in a local maximum for the hill-climber. Use an example.

(c) Is it possible for it to get stuck with convex obstacles?

(d) Would simulated annealing always escape local maxima on this family of problems? Explain!

Exercise 2.3 (Search algorithms)
Prove each of the following statements:

(a) Breadth-first search is a special case of uniform-cost search.

(b) Breadth-first search, depth-first search, and uniform-cost search are special cases of best-first search.

(c) Uniform-cost search is a special case of A* search.

Exercise 2.4 (Board Games)

(a) Consider the game tree for the two-person board game depicted below.
Simulate the behavior of the Minimax algorithm with α-β pruning (always expand children from left to right). Enter the computed node values into the triangles and all intermediate α-β values into the appropriate tables.
(b) Consider the problem of search in a three-player game (you may assume that no alliances are allowed) without the zero-sum condition. The players are called 1, 2, and 3. Unlike in the case of two-player zero-sum games, the evaluation function now returns a triple \((x_1, x_2, x_3)\) such that \(x_i\) is the value the node has for player \(i\).

Complete the game tree given below by annotating all interior nodes and the root node with the backed-up value triples.

```
Sp. 1
Sp. 2
Sp. 3
(1, 2, 3) (4, 2, 1) (6, 1, 2) (7, 4, -1) (5, -1, -1) (-1, 5, 2) (7, 7, -1) (5, 4, 5)
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(c) Assume that the value triple \((5, 4, 5)\) at the rightmost leaf node is replaced by \((5, 4, -1)\). Which problem arises now when you try to back up value triples? Suggest how to modify the back-up procedure to obtain a “robust” result at the root node.

**Exercise 2.5** (Forward Checking / Arc consistency)

Consider the 6-queens problem, where 6 pieces have to be placed on a size 6 \(\times\) 6 board in such a way that no two queens are on the same horizontal, vertical or diagonal line. Let the domains be \(\text{dom}(v_i) = 1, \ldots, 6\) for all variables \(v_i \in V\). Consider now state \(\alpha = \{v_1 \mapsto 2, v_2 \mapsto 5\}\).

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(a) Enforce arc consistency in \(\alpha\). Specify in particular the domains of the variables before and after applying arc consistency. You may assume that the domain of variables with allocated values only consists of that value, while the values of unassigned variables still range over the complete domain. Always choose the variable with lowest index, for which arc consistency has not been established yet.

(b) Apply forward-checking in \(\alpha\). Compare with the result of (a).

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names on your solution.