Foundations of Artificial Intelligence

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Exercise Sheet 3
Due: Wednesday, May 29, 2019, before 12:00

Exercise 3.1 (Satisfiability, Models)

(a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.

    (1) $Smoke \Rightarrow Smoke$

    (2) $Smoke \Rightarrow Fire$

    (3) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Fire \Rightarrow \neg Smoke)$

    (4) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$

    (5) $Spring \Leftrightarrow SunnyWeather$

(b) Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following formulae? Explain.

    (1) $(A \wedge B) \vee (B \wedge C)$

    (2) $A \vee B$

    (3) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 3.2 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ, and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \tag{1}$$
$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \tag{2}$$
$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \tag{3}$$
$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \tag{4}$$
$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \tag{5}$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.

Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show whether $K \models (\neg B \rightarrow (A \wedge C))$ holds.

Exercise 3.3 (Modeling, Proofs)

Consider the following knowledge base:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Using this knowledge base, can you prove that the unicorn is (a) mythical, (b) magical or (c) horned? First, formalize the knowledge base with propositional logic. If a statement is valid or unsatisfiable, use resolution for prove. Else, write down one satisfying and one unsatisfying interpretation.

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names on your solution.