Foundations of Artificial Intelligence

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Exercise Sheet 5

Due: Wednesday, July 3, 2019, before 12:00

Exercise 5.1 (Conditional Independence)

Suppose you are given a bag containing n unbiased coins, out of which $n - 1$ are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

(a) Suppose you reach into the bag, pick out a coin uniformly at random, toss it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?

(b) Suppose you continue flipping the coin for a total of k times after picking it and see k heads. Now what is the conditional probability that you picked the fake coin?

(c) Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure returns FAKE if all k flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error?

Exercise 5.2 (Bayes Rules)

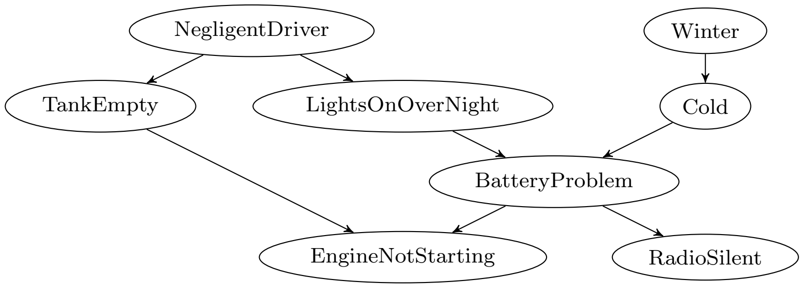
In Freiburg 80% of all cars are red. You see a car at night that does *not* appear red to you. You know that you can correctly identify a red car only in 70% of all cases, given the car is red. However you identify a non-red car correctly in 90% of the cases.

(a) List all conditional and non-conditional probabilities that you can determine directly from the task description. Note: Differentiate between the statement that a car *is red* and the statement that you have *seen a red car*.

(b) Compute the probability that the car is actually red, wenn you perceive a car as red in Freiburg at night.

Exercise 5.3 (Bayesian Networks)

Consider the following Bayesian network:



(a) Determine which of the following conditional independence statements follow from the structure of the Bayesian network ($Ind(U, V \mid W)$ denotes that U is conditionally independent of V given W, and $Ind(U, V)$ denotes unconditional independence of U and V):

(i) $Ind(Cold, Winter)$

(ii) $Ind(Winter, NegligentDriver)$

(iii) $Ind(Winter, RadioSilent \mid BatteryProblem)$

(iv) $Ind(Winter, EngineNotStarting \mid BatteryProblem)$

(v) $Ind(Cold, NegligentDriver \mid RadioSilent)$

(b) Compute $P(EngineNotStarting|NegligentDriver, \neg Cold)$. The relevant entries in the conditional probability tables are given below:

$$P(LightsOnOverNight|NegligentDriver) = 0.3$$
$$P(LightsOnOverNight|\neg NegligentDriver) = 0.02$$
$$P(TankEmpty|NegligentDriver) = 0.1$$
$$P(TankEmpty|\neg NegligentDriver) = 0.01$$
$$P(BatteryProblem|Cold, LightsOnOverNight) = 0.9$$
$$P(BatteryProblem|Cold, \neg LightsOnOverNight) = 0.2$$
$$P(BatteryProblem|\neg Cold, LightsOnOverNight) = 0.8$$
$$P(BatteryProblem|\neg Cold, \neg LightsOnOverNight) = 0.01$$
$$P(EngineNotStarting|BatteryProblem, TankEmpty) = 0.9$$
$$P(EngineNotStarting|BatteryProblem, \neg TankEmpty) = 0.7$$
$$P(EngineNotStarting|\neg BatteryProblem, TankEmpty) = 0.8$$
$$P(EngineNotStarting|\neg BatteryProblem, \neg TankEmpty) = 0.05$$

(c) List all nodes in the Markov blanket for node *LightsOnOverNight*.

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names on your solution.