Foundations of Artificial Intelligence

4. Informed Search Methods

Heuristics, Local Search Methods, Genetic Algorithms

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Contents

Lecture Overview

1. Best-First Search
2. A^* and IDA^*
3. Local Search Methods
4. Genetic Algorithms

Best-First Search

Search procedures differ in the way they determine the next node to expand.

Uninformed Search: Rigid procedure with no knowledge of the cost of a given node to the goal.

Informed Search: Knowledge of the worth of expanding a node n is given in the form of an *evaluation function* $f(n)$, which assigns a real number to each node. Mostly, $f(n)$ includes as a component a *heuristic function* $h(n)$, which estimates the costs of the cheapest path from n to the goal.

$f(n) = g(n) + h(n)$

Best-First Search: Informed search procedure that expands the node with the "best" f-value first.

General Algorithm

function TREE-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    expand the chosen node, adding the resulting nodes to the frontier

Best-first search is an instance of the general TREE-SEARCH algorithm in which *frontier* is a priority queue ordered by an evaluation function f.

When f is always correct, we do not need to search!

Greedy Search

A possible way to judge the “worth” of a node is to estimate its path-costs to the goal.

$$h(n) = \textit{estimated path-costs from } n \textit{ to the goal}$$

The only real restriction is that $h(n) = 0$ if n is a goal.

A best-first search using $h(n)$ as the evaluation function, i.e., $f(n) = h(n)$ is called a *greedy search*.

Example: route-finding problem:
$h(n) =$

Greedy Search

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Example: route-finding problem:
$h(n) =$ straight-line distance from n to the goal

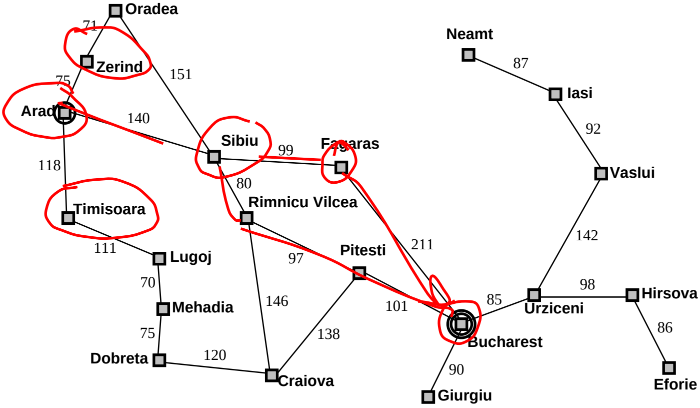
Heuristics

The evaluation function h in greedy searches is also called a *heuristic function* or simply a *heuristic*.

- The word *heuristic* is derived from the Greek word $\varepsilon\upsilon\rho\iota\sigma\kappa\varepsilon\iota\nu$ (note also: $\varepsilon\upsilon\rho\eta\kappa\alpha$!)
- The mathematician Polya introduced the word in the context of problem solving techniques.
- In AI it has two meanings:
  - Heuristics are fast but in certain situations incomplete methods for problem-solving [Newell, Shaw, Simon 1963] (The greedy search is actually generally incomplete).
  - Heuristics are methods that improve the search in the average-case.

→ In all cases, the heuristic is *problem-specific* and *focuses* the search!

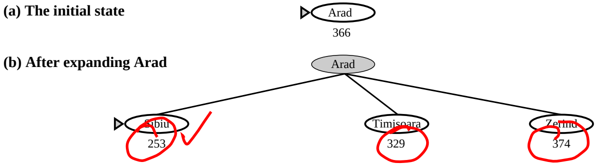
Greedy Search Example



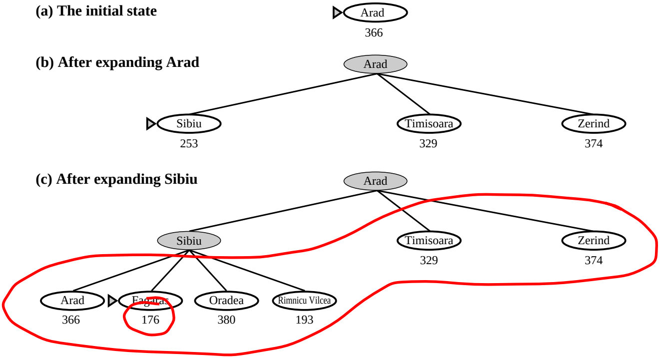
Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Search from Arad to Bucharest

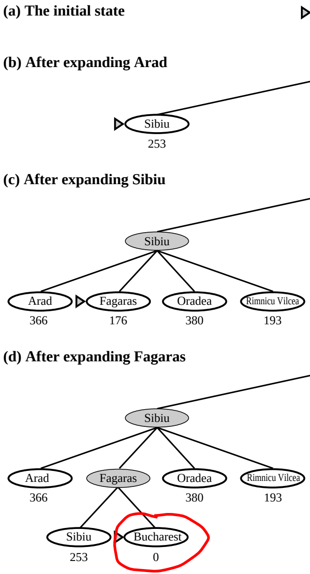
Greedy Search from Arad to Bucharest



Greedy Search from Arad to Bucharest



Greedy Search from Arad to Bucharest



Greedy Search - Properties

- a good heuristic might reduce search time drastically
- non-optimal
- incomplete
- graph-search version is complete only in finite spaces

Can we do better?

Lecture Overview

A*: Minimization of the Estimated Path Costs

A* combines greedy search with the uniform-cost search:
Always expand node with lowest $f(n)$ first, where

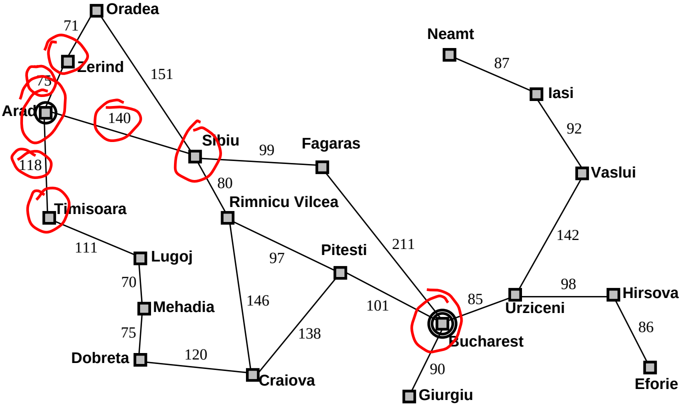
$g(n)$ = actual cost from the initial state to n.
$h(n)$ = estimated cost from n to the nearest goal.
$f(n) = g(n) + h(n)$,
the estimated cost of the cheapest solution through n.

Let $h^*(n)$ be the actual cost of the optimal path from n to the nearest goal. h is *admissible* if the following holds for all n:

$$h(n) \leq h^*(n)$$

We require that for A*, h is admissible (example: straight-line distance is admissible).
In other words, h is an *optimistic* estimate of the costs that actually occur.

A* Search Example



Arad	366
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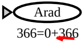
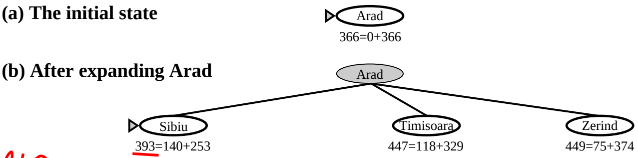
Urziceni	80
Vaslui	199
Zerind	374

A^* Search from Arad to Bucharest

(a) The initial state

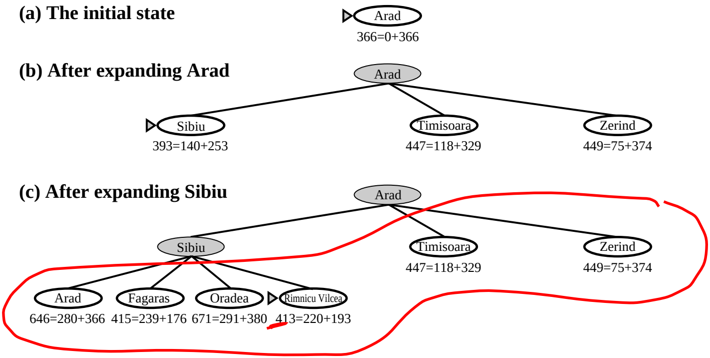




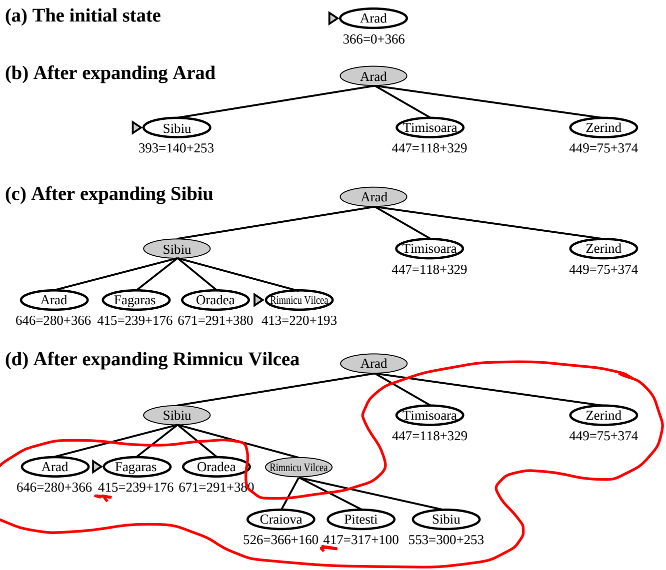
(a) The initial state
Arad
366=0+366
(b) After expanding Arad
Arad
Sibiu
393=140+253
Timisoara
447=118+329
Zerind
449=75+374

$f(n) = 140 + 253$

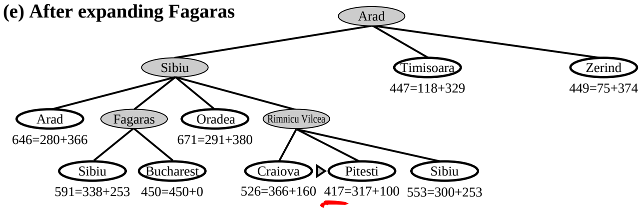
A* Search from Arad to Bucharest



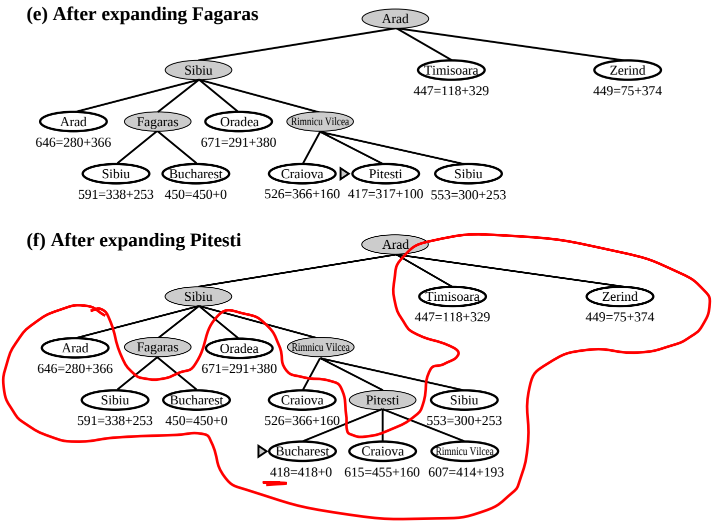
A* Search from Arad to Bucharest



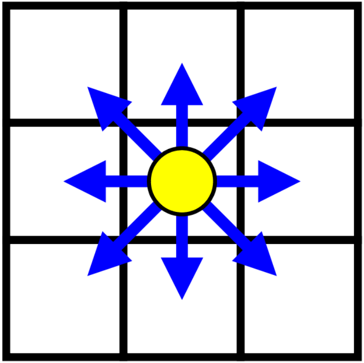
A* Search from Arad to Bucharest

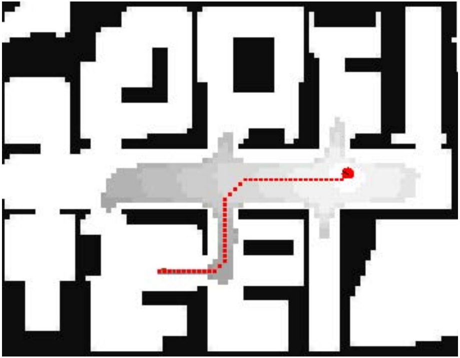


A* Search from Arad to Bucharest



Example: Path Planning for Robots in a Grid-World



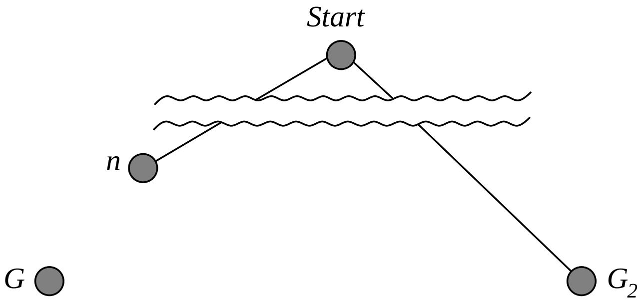


Live-Demo: `http://qiao.github.io/PathFinding.js/visual/`

Optimality of A*

Claim: The first solution found (= node is expanded and found to be a goal node) has the minimum path cost.

Proof: Suppose there exists a goal node G with optimal path cost f^*, but A* has found another node G_2 with $g(G_2) > f^*$.



Optimality of A*

Let n be a node on the path from the start to G that has not yet been expanded. Since h is admissible, we have

$$f(n) \leq f^*.$$

Since n was not expanded before G_2, the following must hold:

$$f(G_2) \leq f(n)$$

and

$$f(G_2) \leq f^*.$$

It follows from $h(G_2) = 0$ that

$$g(G_2) \leq f^*.$$

$\rightarrow$ Contradicts the assumption!

Completeness and Complexity

Completeness:

If a solution exists, A* will find it provided that (1) every node has a finite number of successor nodes, and (2) there exists a positive constant $\delta > 0$ such that every step has at least cost δ.

$\rightarrow$ there exists only a finite number of nodes n with $f(n) \leq f^*$.

Complexity:

In general, still exponential in the path length of the solution (space, time)

More refined complexity results depend on the assumptions made, e.g. on the quality of the heuristic function. Example:

In the case in which $|h^*(n) - h(n)| \leq O(\log(h^*(n))$, only one goal state exists, and the search graph is a tree, a sub-exponential number of nodes will be expanded [Gaschnig, 1977, Helmert & Roeger, 2008]. Unfortunately, this almost never holds.

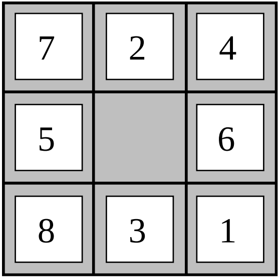
A note on Graph- vs. Tree-Search

- A* as described is a tree-search (and may consider duplicates)
- For the graph-based variant, one
  - either needs to consider re-opening nodes from the *explored* set, when a better estimate becomes known, or
  - one needs needs to require stronger restrictions on the heuristic estimate: it needs to be consistent.

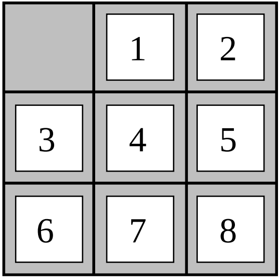
→ A heuristic h is called **consistent** iff for all actions a leading from s to s': $h(s) - h(s') \leq c(a)$, where $c(a)$ denotes the cost of action a.

- **Note**: Consistency implies admissibility.

Heuristic Function Example

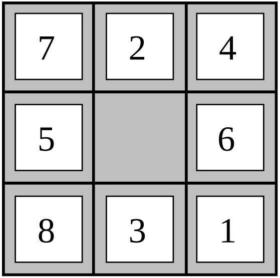


Start State

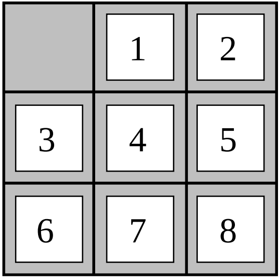


Goal State

Heuristic Function Example



Start State



Goal State

h_1 = the number of tiles in the wrong position

Heuristic Function Example

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

h_1 = the number of tiles in the wrong position

h_2 = the sum of the distances of the tiles from their goal positions (*Manhattan distance*)

Empirical Evaluation

- d = distance from goal
- Average over 100 instances

	Search Cost (nodes generated)			Effective Branching Factor		
d	IDS	A$^*(h_1)$	A$^*(h_2)$	IDS	A$^*(h_1)$	A$^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	-	539	113	-	1.44	1.23
16	-	1301	211	-	1.45	1.25
18	-	3056	363	-	1.46	1.26
20	-	7276	676	-	1.47	1.47
22	-	18094	1219	-	1.48	1.28
24	-	39135	1641	-	1.48	1.26

Variants of A*

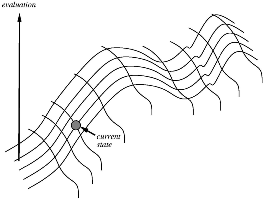
A* in general still suffers from exponential memory growth. Therefore, several refinements have been suggested:

- iterative-deepening A*, where the f-costs are used to define the cutoff (rather than the depth of the search tree): **IDA***
- Recursive Best First Search (RBFS): introduces a variable f_limit to keep track of the best alternative path available from any ancestor of the current node. If current node exceeds this limit, recursion unwinds back to the alternative path.
- other alternatives memory-bounded A* (MA*) and simplified MA* (SMA*).

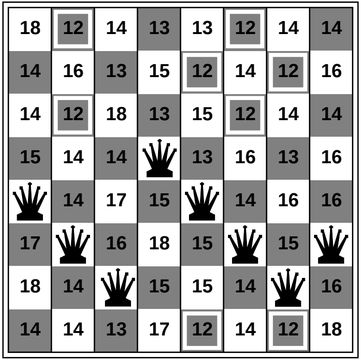
Lecture Overview

Local Search Methods

- In many problems, it is unimportant how the goal is reached—only the goal itself matters (8-queens problem, VLSI Layout, TSP).
- If in addition a quality measure for states is given, **local search** can be used to find solutions.
- It operates using a single current node (rather than multiple paths).
- It requires little memory.
- **Idea**: Begin with a randomly-chosen configuration and improve on it step by step → **Hill Climbing**.
- **Note**: It can be used for maximization or minimization respectively (see 8-queens example)



Example state with heuristic cost estimate $h = 17$ (counts the number of pairs threatening each other directly or indirectly).



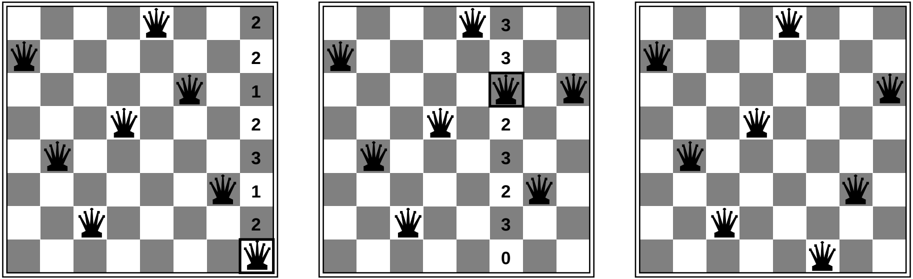
Hill Climbing

function HILL-CLIMBING($problem$) **returns** a state that is a local maximum

$current \leftarrow$ MAKE-NODE($problem$.INITIAL-STATE)
loop do
$neighbor \leftarrow$ a highest-valued successor of $current$
if neighbor.VALUE $\leq$ current.VALUE **then return** $current$.STATE
$current \leftarrow neighbor$

Example: 8-queens Problem (2)

Possible realization of a hill-climbing algorithm:
Select a column and move the queen to the square with the fewest conflicts.



Problems with Local Search Methods

- *Local maxima:* The algorithm finds a sub-optimal solution.
- *Plateaus:* Here, the algorithm can only explore at random.
- Ridges: Similar to plateaus but might even require suboptimal moves.

Solutions:

- *Start over* when no progress is being made.
- "Inject noise" $\rightarrow$ random walk

Which strategies (with which parameters) are successful (within a problem class) can usually only empirically be determined.

Example: 8-queens Problem (Local Minimum)

Local minimum ($h = 1$) of the 8-queens Problem. Every successor has a higher cost.

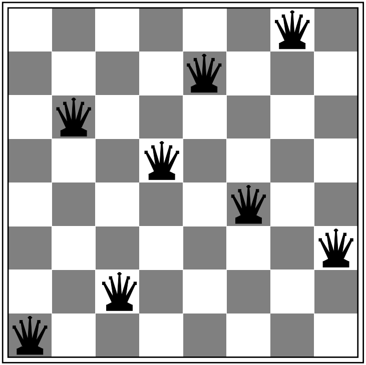
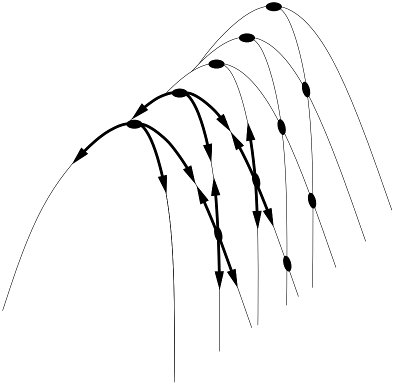


Illustration of the ridge problem

The grid of states (dark circles) is superimposed on a ridge rising from left to right, creating a sequence of local maxima, that are not directly connected to each other. From each local maximum, all the available actions point downhill.



Performance figures for the 8-queens Problem

The 8-queens problem has about $8^8 \approx 17\ million$ states. Starting from a random initialization, hill-climbing directly finds a solution in about 14% of the cases. On average it requires only 4 steps!

Better algorithm: Allow sideways moves (no improvement), but restrict the number of moves (avoid infinite loops!).

E.g.: max. 100 moves: Solves 94%, number of steps raises to 21 steps for successful instances and 64 for failure cases.

Simulated Annealing

In the simulated annealing algorithm, “noise” is injected systematically: first a lot, then gradually less.

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to “temperature”

   current ← MAKE-NODE(problem.INITIAL-STATE)
   for t = 1 to ∞ do
      T ← schedule(t)
      if T = 0 then return current
      next ← a randomly selected successor of current
      ΔE ← next.VALUE – current.VALUE
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(ΔE/T)
```

Has been used since the early 80’s for VSLI layout and other optimization problems.

Lecture Overview

Genetic Algorithms

Evolution appears to be very successful at finding good solutions.

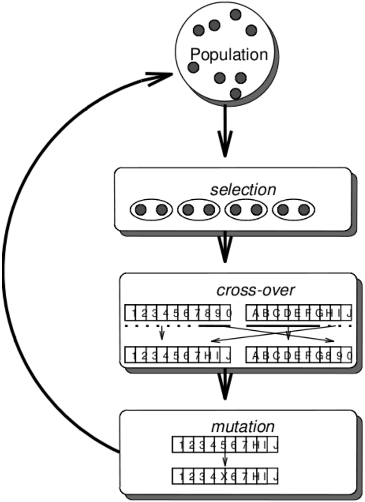
Idea: Similar to evolution, we search for solutions by three operators: "mutation", "crossover", and "selection".

Ingredients:

- Coding of a solution into a string of symbols or bit-string
- A fitness function to judge the worth of configurations
- A population of configurations

Example: Represent an *individual* (one 8-queens configuration) as a concatenation of eight x-y coordinates. Its fitness is judged by the number of non-attacks. The *population* consists of a set of configurations.

Selection, Mutation, and Crossing



Many variations:

how selection will be applied, what type of cross-over operators will be used, etc.

Selection of individuals according to a fitness function and pairing

Calculation of the breaking points and recombination

According to a given probability elements in the string are modified.

Summary

- Heuristics focus the search
- Best-first search expands the node with the highest worth (defined by any measure) first.
- With the minimization of the evaluated costs to the goal h we obtain a greedy search.
- The minimization of $f(n) = g(n) + h(n)$ combines uniform and greedy searches. When $h(n)$ is admissible, i.e., h^* is never overestimated, we obtain the A* search, which is complete and optimal.
- IDA* is a combination of the iterative-deepening and A* searches.
- Local search methods only ever work on one state, attempting to improve it step-wise.
- Genetic algorithms imitate evolution by combining good solutions.