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Lecture Overview

1. Best-First Search

2. A* and IDA*

3. Local Search Methods

4. Genetic Algorithms
Search procedures differ in the way they determine the next node to expand.

**Uninformed Search:** Rigid procedure with no knowledge of the cost of a given node to the goal.

**Informed Search:** Knowledge of the worth of expanding a node $n$ is given in the form of an evaluation function $f(n)$, which assigns a real number to each node. Mostly, $f(n)$ includes as a component a heuristic function $h(n)$, which estimates the costs of the cheapest path from $n$ to the goal.

$$f(n) = g(n) + h(n)$$

**Best-First Search:** Informed search procedure that expands the node with the “best” $f$-value first.
function TREE-SEARCH(problem) returns a solution, or failure
initialize the frontier using the initial state of problem
loop do
    if the frontier is empty then return failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state then return the corresponding solution
    expand the chosen node, adding the resulting nodes to the frontier

Best-first search is an instance of the general TREE-SEARCH algorithm in which frontier is a priority queue ordered by an evaluation function $f$. When $f$ is always correct, we do not need to search!
Greedy Search

A possible way to judge the “worth” of a node is to estimate its path-costs to the goal.

\[ h(n) = \text{estimated path-costs from } n \text{ to the goal} \]

The only real restriction is that \( h(n) = 0 \) if \( n \) is a goal.

A best-first search using \( h(n) \) as the evaluation function, i.e., \( f(n) = h(n) \), is called a greedy search.

Example: route-finding problem:
\[ h(n) = \]
A possible way to judge the “worth” of a node is to estimate its path-costs to the goal.

\[ h(n) = \text{estimated path-costs from } n \text{ to the goal} \]

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A best-first search using \( h(n) \) as the evaluation function, i.e., \( f(n) = h(n) \) is called a greedy search.

Example: route-finding problem:

\[ h(n) = \text{straight-line distance from } n \text{ to the goal} \]
The evaluation function $h$ in greedy searches is also called a **heuristic** function or simply a **heuristic**.

- The word *heuristic* is derived from the Greek word $ευρισκειν$ (note also: $ευρηκα$!)
- The mathematician Polya introduced the word in the context of problem solving techniques.
- In AI it has two meanings:
  - Heuristics are fast but in certain situations incomplete methods for problem-solving [Newell, Shaw, Simon 1963] (The greedy search is actually generally incomplete).
  - Heuristics are methods that improve the search in the average-case.

→ In all cases, the heuristic is *problem-specific* and *focuses* the search.
Greedy Search Example
Greedy Search from Arad to Bucharest

(a) The initial state

- Arad

(b) After expanding Arad

- Arad

(c) After expanding Sibiu

- Sibiu

(d) After expanding Fagaras

- Fagaras

- Rimnicu Vilcea

- Zerind

- Arad

- Sibiu

- Arad

- Fagaras

- Oradea

- Timisoara

- Sibiu

- Bucharest

- 329

- 374

- 366

- 380

- 193

- 253

- 0

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Foundations of AI

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Greedy Search from Arad to Bucharest

(a) The initial state

(b) After expanding Arad
Greedy Search from Arad to Bucharest

(a) The initial state

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Greedy Search - Properties

- a good heuristic might reduce search time drastically
- non-optimal
- incomplete
- graph-search version is complete only in finite spaces

Can we do better?
Lecture Overview

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A*: Minimization of the Estimated Path Costs

**A**\(^\ast\) combines greedy search with the uniform-cost search: Always expand node with lowest \( f(n) \) first, where

\[
g(n) = \text{actual cost from the initial state to } n.
\]

\[
h(n) = \text{estimated cost from } n \text{ to the nearest goal.}
\]

\[
f(n) = g(n) + h(n),
\]

the estimated cost of the cheapest solution through \( n \).

Let \( h^\ast(n) \) be the actual cost of the optimal path from \( n \) to the nearest goal. \( h \) is *admissible* if the following holds for all \( n \):

\[
h(n) \leq h^\ast(n)
\]

We require that for \( \text{A}^\ast \), \( h \) is admissible (example: straight-line distance is admissible).

In other words, \( h \) is an *optimistic* estimate of the costs that actually occur.
A* Search from Arad to Bucharest

(a) The initial state

- **Arad**
  - 366 = 0 + 366
A* Search from Arad to Bucharest

(a) The initial state

(b) After expanding Arad

\[ f(n) = h(n) + g(n) \]

\[ f(n) = 140 + 253 \]
A* Search from Arad to Bucharest

(a) The initial state

(b) After expanding Arad

(c) After expanding Sibiu
A* Search from Arad to Bucharest

(a) The initial state

(b) After expanding Arad

(c) After expanding Sibiu

(d) After expanding Rimnicu Vilcea
A* Search from Arad to Bucharest

(e) After expanding Fagaras

- **Arad**
  - **Sibiu**: 447 = 118 + 329
  - **Timisoara**: 449 = 75 + 374
  - **Zerind**: 449 = 75 + 374

- **Sibiu**
  - **Zerind**: 449 = 75 + 374
  - **Arad**: 366 = 0 + 366
  - **Timisoara**: 447 = 118 + 329

- **Arad**
  - **Sibiu**: 447 = 118 + 329
  - **Timisoara**: 447 = 118 + 329
  - **Zerind**: 449 = 75 + 374

- **Rimnicu Vilcea**
  - **Sibiu**: 526 = 366 + 160
  - **Fagaras**: 646 = 280 + 366
  - **Oradea**: 591 = 338 + 253

- **Fagaras**
  - **Oradea**: 553 = 300 + 253
  - **Pitesti**: 415 = 239 + 176
  - **Sibiu**: 553 = 300 + 253

- **Oradea**
  - **Craiova**: 615 = 455 + 160
  - **Bucharest**: 646 = 280 + 366
  - **Sibiu**: 591 = 338 + 253

- **Craiova**
  - **Bucharest**: 646 = 280 + 366
  - **Sibiu**: 591 = 338 + 253

- **Pitesti**
  - **Sibiu**: 553 = 300 + 253
  - **Sibiu**: 553 = 300 + 253

- **Bucharest**
  - **Sibiu**: 591 = 338 + 253
  - **Fagaras**: 646 = 280 + 366
  - **Oradea**: 591 = 338 + 253

- **Fagaras**
  - **Oradea**: 553 = 300 + 253
  - **Pitesti**: 415 = 239 + 176

- **Timisoara**
  - **Sibiu**: 447 = 118 + 329
  - **Arad**: 366 = 0 + 366

- **Zerind**
  - **Sibiu**: 449 = 75 + 374

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Foundations of AI

May 8, 2019
A* Search from Arad to Bucharest

(e) After expanding Fagaras

(f) After expanding Pitesti
Example: Path Planning for Robots in a Grid-World

Live-Demo: http://qiao.github.io/PathFinding.js/visual/
**Claim:** The first solution found (= node is expanded and found to be a goal node) has the minimum path cost.

**Proof:** Suppose there exists a goal node $G$ with optimal path cost $f^*$, but $A^*$ has first found another node $G_2$ with $g(G_2) > f^*$. 

Optimality of $A^*$

\[ f(n) = g(n) + h(n) \]
Let $n$ be a node on the path from the start to $G$ that has not yet been expanded. Since $h$ is admissible, we have

$$f(n) \leq f^*.$$ 

Since $n$ was not expanded before $G_2$, the following must hold:

$$f(G_2) \leq f(n)$$

and

$$f(G_2) \leq f^*.$$ 

It follows from $h(G_2) = 0$ that

$$g(G_2) \leq f^*.$$ 

→ Contradicts the assumption!
**Completeness:**

If a solution exists, $A^*$ will find it provided that (1) every node has a finite number of successor nodes, and (2) there exists a positive constant $\delta > 0$ such that every step has at least cost $\delta$.

$\rightarrow$ there exists only a finite number of nodes $n$ with $f(n) \leq f^*$.

**Complexity:**

In general, still exponential in the path length of the solution (space, time).

More refined complexity results depend on the assumptions made, e.g. on the quality of the heuristic function. Example:

In the case in which $|h^*(n) - h(n)| \leq O(\log(h^*(n)))$, only one goal state exists, and the search graph is a tree, a sub-exponential number of nodes will be expanded [Gaschnig, 1977, Helmert & Roeger, 2008]. Unfortunately, this almost never holds.
A note on Graph- vs. Tree-Search

- A* as described is a tree-search (and may consider duplicates)
- For the graph-based variant, one
  - either needs to consider re-opening nodes from the explored set, when a better estimate becomes known, or
  - one needs to require stronger restrictions on the heuristic estimate: it needs to be consistent.

→ A heuristic $h$ is called **consistent** iff for all actions $a$ leading from $s$ to $s'$: $h(s) - h(s') \leq c(a)$, where $c(a)$ denotes the cost of action $a$. (Consistent heuristics prevent the need to re-open nodes from the explored set.)

- **Note:** Consistency implies admissibility.
- **Note:** A* can still be applied if heuristic is not consistent, but optimality is lost in this case.
Heuristic Function Example

- **Start State**:
  - 7
  - 2
  - 4
  - 5
  - 6
  - 3
  - 1
  - 8

- **Goal State**:
  - 1
  - 2
  - 3
  - 4
  - 5
  - 6
  - 7
  - 8

- $h_1$ = the number of tiles in the wrong position
- $h_2$ = the sum of the distances of the tiles from their goal positions (Manhattan distance)
$h_1 =$ the number of tiles in the wrong position
Heuristic Function Example

\[ h_1 = \text{the number of tiles in the wrong position} \]
\[ h_2 = \text{the sum of the distances of the tiles from their goal positions} \]

\textit{(Manhattan distance)}
$d = \text{distance from goal}$

- Average over 100 instances

<table>
<thead>
<tr>
<th>$d$</th>
<th>Search Cost (nodes generated)</th>
<th>Effective Branching Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IDS</td>
<td>$A^*(h_1)$</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>680</td>
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</tr>
<tr>
<td>8</td>
<td>6384</td>
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<tr>
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<td>93</td>
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<td>12</td>
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<td>14</td>
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<td>22</td>
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<td>18094</td>
</tr>
<tr>
<td>24</td>
<td>-</td>
<td>39135</td>
</tr>
</tbody>
</table>
Variants of A*

A* in general still suffers from exponential memory growth. Therefore, several refinements have been suggested:

- **iterative-deepening A***, where the f-costs are used to define the cutoff (rather than the depth of the search tree): \( \text{IDA}\).

- **Recursive Best First Search (RBFS)**: introduces a variable \( f\_\text{limit} \) to keep track of the best alternative path available from any ancestor of the current node. If the current node exceeds this limit, recursion unwinds back to the alternative path.

- Other alternatives: memory-bounded A* (MA*) and simplified MA* (SMA*).
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In many problems, it is unimportant how the goal is reached—only the goal itself matters (8-queens problem, VLSI Layout, TSP).

If in addition a quality measure for states is given, **local search** can be used to find solutions.

It operates using a single current node (rather than multiple paths).

It requires little memory (cp. to most systematic search strategies!)

**Idea**: Begin with a randomly-chosen configuration and improve on it step by step → **Hill Climbing / Gradient Decent**.

**Note**: It can be used for maximization or minimization respectively (see 8-queens example)
Example: 8-queens Problem (1)

Example state with heuristic cost estimate $h = 17$ (counts the number of pairs threatening each other directly or indirectly).
function HILL-CLIMBING(problem) returns a state that is a local maximum

current ← MAKE-NODE(problem.INITIAL-STATE)

loop do
    \textit{neighbor} ← a highest-valued successor of \textit{current}
    if \textit{neighbor}.VALUE \leq \textit{current}.VALUE then return \textit{current}.STATE
    current ← \textit{neighbor}
Possible realization of a hill-climbing algorithm:
Select a column and move the queen to the square with the fewest conflicts.
Problems with Local Search Methods

- **Local maxima**: The algorithm finds a sub-optimal solution.
- **Plateaus**: Here, the algorithm can only explore at random.
- Ridges: Similar to plateaus but might even require suboptimal moves.

**Solutions:**
- *Start over* when no progress is being made.
- “Inject noise” → random walk

Which strategies (with which parameters) are successful (within a problem class) can usually only empirically be determined.
Local minimum \((h = 1)\) of the 8-queens Problem. Every successor has a higher cost.
Illustration of the ridge problem

The grid of states (dark circles) is superimposed on a ridge rising from left to right, creating a sequence of local maxima, that are not directly connected to each other. From each local maximum, all the available actions point downhill.
The 8-queens problem has about $8^8 \approx 17$ million states. Starting from a random initialization, hill-climbing directly finds a solution in about 14% of the cases. On average it requires only 4 steps!

Better algorithm: Allow sideways moves (no improvement), but restrict the number of moves (avoid infinite loops!).

E.g.: max. 100 moves: Solves 94%, number of steps raises to 21 steps for successful instances and 64 for failure cases.
Simulated Annealing

In the simulated annealing algorithm, “noise” is injected systematically: first a lot, then gradually less.

```plaintext
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
inputs: problem, a problem
         schedule, a mapping from time to "temperature"

current ← MAKE-NODE(problem.INITIAL-STATE)
for t = 1 to ∞ do
    T ← schedule(t)
    if T = 0 then return current
    next ← a randomly selected successor of current
    ΔE ← next.VALUE - current.VALUE
    if ΔE > 0 then current ← next
    else current ← next only with probability e^{ΔE/T}
```

Has been used since the early 80’s for VSLI layout and other optimization problems.
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Evolution appears to be very successful at finding good solutions.

**Idea:** Similar to evolution, we search for solutions by three operators: “mutation”, “crossover”, and “selection”.

**Ingredients:**
- Coding of a solution into a string of symbols or bit-string
- A fitness function to judge the worth of configurations
- A population of configurations

**Example:** Represent an individual (one 8-queens configuration) as a concatenation of eight x-y coordinates. Its fitness is judged by the number of non-attacks. The population consists of a set of configurations.
Selection, Mutation, and Crossing

Many variations:

how selection will be applied, what type of cross-over operators will be used, etc.

Selection of individuals according to a fitness function and pairing

Calculation of the breaking points and recombination

According to a given probability elements in the string are modified.
Summary

- **Heuristics** focus the search
- **Best-first search** expands the node with the highest worth (defined by any measure) first.
- With the minimization of the evaluated costs to the goal $h$ we obtain a **greedy search**.
- The minimization of $f(n) = g(n) + h(n)$ combines uniform and greedy searches. When $h(n)$ is admissible, i.e., $h^*$ is never overestimated, we obtain the **A* search**, which is complete and optimal.
- **IDA* is a combination of the iterative-deepening and A* searches.**
- **Local search methods** work on a single state only, attempting to improve it step-wise.
- **Genetic algorithms** imitate evolution by combining good solutions.