Foundations of Artificial Intelligence

8. Satisfiability and Model Construction

DPLL Procedure, Phase Transitions, Local Search, State of the Art

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Motivation

SAT solving is the best available technology for practical solutions to many NP-hard problems

- Formal verification
  - Verification of software
    - Ruling out unintended states (null-pointer exceptions, etc.)
    - Proving that the program computes the right solution
  - Verification of hardware (Pentium bug, etc)
- Practical approach:
  encode into SAT & exploit the rapid progress in SAT solving
  - Solving CSP instances in practice
  - Solving graph coloring problems in practice
  - ...

Contents

Lecture Overview

Logical deduction vs. satisfiability

Propositional Logic — typical algorithmic questions:

Logical deduction

Given: A logical theory (set of propositions)

Question: Does a proposition logically follow from this theory?

Reduction to unsatisfiability, which is coNP-complete (complementary to NP problems)

Satisfiability of a formula (SAT)

Given: A logical theory

Wanted: Model of the theory

Example: Configurations that fulfill the constraints given in the theory

Can be “easier” because it is enough to find one model

The Satisfiability Problem (SAT)

Given:

Propositional formula φ in CNF

Wanted:

Model of φ.

or proof, that no such model exists.

SAT and CSP

SAT can be formulated as a Constraint-Satisfaction-Problem ($\rightarrow$ search):

SAT and CSP

SAT can be formulated as a Constraint-Satisfaction-Problem ($\rightarrow$ search):

- CSP-Variables = Symbols of the alphabet
- Domain of values = $\{T, F\}$
- Constraints given by clauses

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1. The SAT Problem
2. Davis-Putnam-Logemann-Loveland (DPLL) Procedure
3. "Average" Complexity of the Satisfiability Problem
4. Local Search Procedures
5. State of the Art

The DPLL algorithm

The DPLL algorithm (Davis, Putnam, Logemann, Loveland, 1962) corresponds to backtracking with inference in CSPs:

- Recursive call DPLL (Δ, l) with
  - Δ: set of clauses
  - l: partial variable assignment
- Result: satisfying assignment that extends l or "unsatisfiable" if no such assignment exists.
- First call by DPLL$(\Delta, \emptyset)$

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Inference in DPLL:

- Simplify: if variable v is assigned a value d, then all clauses containing v are simplified immediately (corresponds to forward checking)
- Variables in unit clauses ($=$ clauses with only one variable) are immediately assigned (corresponds to minimum remaining values ordering in CSPs)

The DPLL Procedure

DPLL Function

Given a set of clauses Δ defined over a set of variables Σ, return "satisfiable" if Δ is satisfiable. Otherwise return "unsatisfiable".

1. If $\Delta = \emptyset$ return "satisfiable"
2. If $\Box \in \Delta$ return "unsatisfiable"
3. Unit-propagation Rule: If Δ contains a unit-clause C, assign a truth-value to the variable in C that satisfies C, simplify Δ to Δ' and return $\mathrm{DPLL}(\Delta')$.

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4. Splitting Rule: Select from Σ a variable v which has not been assigned a truth-value. Assign one truth value t to it, simplify Δ to Δ' and call $\mathrm{DPLL}(\Delta')$
   a. If the call returns "satisfiable", then return "satisfiable".
   b. Otherwise assign *the other* truth-value to v in Δ, simplify to Δ'' and return $\mathrm{DPLL}(\Delta'')$.

Example (1)

$$\Delta = \{\{a, b, \neg c\}, \{\neg a, \neg b\}, \{c\}, \{a, \neg b\}\}$$

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Properties of DPLL

DPLL is complete, correct, and guaranteed to terminate.

DPLL constructs a model, if one exists.

In general, DPLL requires exponential time (splitting rule!)
$\rightarrow$ *Heuristics* are needed to determine which variable should be instantiated next and which value should be used.

DPLL is polynomial on Horn clauses (see next slides).

In current SAT competitions, DPLL-based procedures have shown the best performance.

DPLL on Horn Clauses (0)

Horn Clauses constitute an important special case, since they require only polynomial runtime of DPLL.

Definition: A Horn clause is a clause with maximally one positive literal
E.g., $\neg A_1 \vee \ldots \vee \neg A_n \vee B$ or $\neg A_1 \vee \ldots \vee \neg A_n$
($n = 0$ is permitted).

Equivalent representation: $\neg A_1 \vee \ldots \vee \neg A_n \vee B \Leftrightarrow \bigwedge_i A_i \Rightarrow B$
$\rightarrow$ Basis of logic programming (e.g., PROLOG)

DPLL on Horn Clauses (1)

Note:

1. The simplifications in DPLL on Horn clauses always generate Horn clauses
2. If the first sequence of applications of the unit propagation rule in DPLL does not lead to termination, a set of Horn clauses without unit clauses is generated
3. A set of Horn clauses without unit clauses and without the empty clause is satisfiable, since

   All clauses have at least one negative literal (since all non-unit clauses have at least two literals, where at most one can be positive (Def. Horn))

   Assigning $false$ to all variables satisfies formula

DPLL on Horn Clauses (2)

4. It follows from 3.:
   a. every time the splitting rule is applied, the current formula is satisfiable
   b. every time, when the wrong decision (= assignment in the splitting rule) is made, this will be immediately detected (e.g., only through unit propagation steps and the derivation of the empty clause).
5. Therefore, the search trees for n variables can only contain a maximum of n nodes, in which the splitting rule is applied (and the tree branches).
6. Therefore, the size of the search tree is only polynomial in n and therefore the running time is also polynomial.

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How Good is DPLL in the Average Case?

We know that SAT is NP-complete, i.e., in the worst case, it takes exponential time.

This is clearly also true for the DPLL-procedure.
$\rightarrow$ Couldn't we do better in the average case?

For CNF-formulae, in which the probability for a positive appearance, negative appearance and non-appearance in a clause is $1/3$, DPLL needs on average quadratic time (Goldberg 79)!
$\rightarrow$ The probability that these formulae are satisfiable is, however, very high.

Phase Transitions . . .

Conversely, we can, of course, try to identify hard to solve problem instances.

Cheeseman et al. (IJCAI-91) came up with the following plausible conjecture:

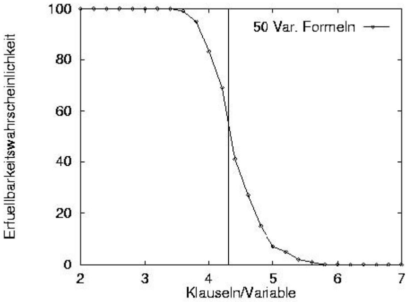
> All NP-complete problems have at least *one order* parameter and the hard to solve problems are around a critical value of this order parameter. This critical value (a phase transition) separates one region from another, such as over-constrained and under-constrained regions of the problem space.

Confirmation for graph coloring and Hamiltonian path . . . , later also for other NP-complete problems.

Phase Transitions with 3-SAT

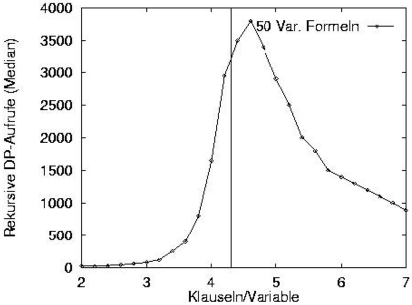
Constant clause length model (Mitchell et al., AAAI-92):
Clause length k is given. Choose variables for every clause k and use the complement with probability 0.5 for each variable.

Phase transition for 3-SAT with a clause/variable ratio of approx. 4.3:



Empirical Difficulty

The Davis-Putnam (DPLL) Procedure shows extreme runtime peaks at the phase transition:



Note: Hard instances can exist even in the regions of the more easily satisfiable/unsatisfiable instances!

Notes on the Phase Transition

When the probability of a solution is close to 1 (under-constrained), there are many solutions, and the first search path of a backtracking search is usually successful.

If the probability of a solution is close to 0 (over-constrained), this fact can usually be determined early in the search.

In the phase transition stage, there are many near successes

$\rightarrow$ (limited) possibility of predicting the difficulty of finding a solution based on the parameters

$\rightarrow$ (search intensive) benchmark problems are located in the phase region (but they have a special structure)

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Local Search Methods for Solving Logical Problems

In many cases, we are interested in finding a satisfying assignment of variables (example CSP), and we can sacrifice completeness if we can "solve" much larger instances this way.

Standard process for optimization problems: Local Search

- Based on a (random) configuration
- Through local modifications, we hope to produce better configurations

$\rightarrow$ Main problem: local maxima

Dealing with Local Maxima

As a measure of the value of a configuration in a logical problem, we could use the number of satisfied constraints/clauses.

At first glance, local search seems inappropriate, considering that we want to find a global maximum (all constraints/clauses satisfied).

However:

- By restarting and/or injecting noise, we can often escape local maxima.
- Local search can perform very well for SAT solving

A pioneering local search method for SAT: GSAT (1993)

Procedure GSAT

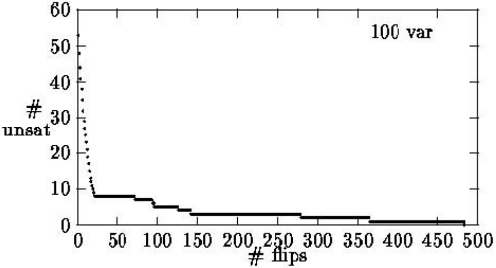
INPUT: a set of clauses α, $\textsc{Max-Flips}$, and $\textsc{Max-Tries}$
OUTPUT: a satisfying truth assignment of α, if found

```
begin
  for i := 1 to MAX-TRIES
    T := a randomly-generated truth assignment
    for j := 1 to MAX-FLIPS
      if T satisfies α then return T
      v := a propositional variable such that a change in its
             truth assignment gives the largest increase in
             the number of clauses of α that are satisfied by T
      T := T with the truth assignment of v reversed
    end for
  end for
  return "no satisfying assignment found"
end
```

The Search Behavior of GSAT

In contrast to many other local search methods, we must also allow sideways movements!

Most time is spent searching on plateaus.



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Practical Improvements of DPLL Algorithms

Clause Learning

- Consider an exemplary SAT problem

  26 variables A, . . . , Z

  Amongst many other clauses, we have

  $\{(\neg A, Y, Z)\}, \{(\neg A, \neg Y, Z)\}, \{(\neg A, Y, \neg Z)\}, \{(\neg A, \neg Y, \neg Z)\}$

  We'll branch on variables in lexicographic order and try true first
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  - There is no satisfying assignment to the clauses above when A=T
  - For each assignment to variables B, . . . , X, we'll have to rediscover this fact
  - Rather: reason about the variables that led to a conflict: A, Y and Z
  - We can 'learn" (here: logically infer) a new clause: $\neg A$
  - Leads to conflict-directed clause learning (CDCL)

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Intelligent Backjumping

- Closely related to clause learning

Practical Improvements of SAT Algorithms

Both for DPLL/CDCL algorithms and local search algorithms

- Randomization and restarts
- Efficient data structures, indexing, etc
- Engineering ingenious heuristics

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Efficient data structures, indexing, etc

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Meta-algorithmic advances

- Automated parameter tuning and algorithm configuration
- Selection of the best-fitting algorithm based on instance characteristics
- Selection of the best-fitting parameters based on instance characteristics
- Use of machine learning to pinpoint what factors most affects performance

The Current State of the Art

SAT competitions since beginning of the 90s

Current SAT competitions (`http://www.satcompetition.org/`):

- Largest "industrial" instances: $> 10{,}000{,}000$ variables

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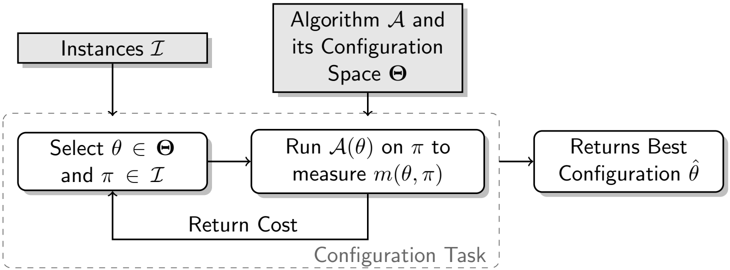
Best solvers use meta-algorithmic methods, such as algorithm configuration, selection, etc.

- We thus discuss these briefly next

Algorithm Configuration



Definition: algorithm configuration

Given:

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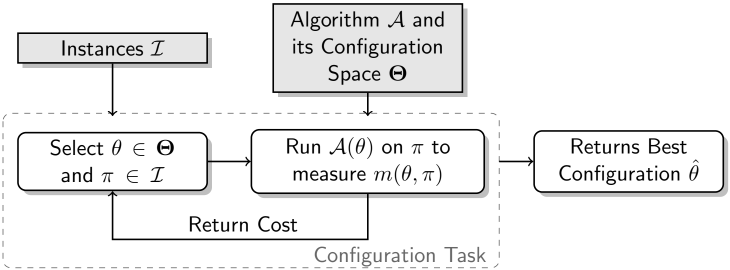

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a cost metric $m : \mathbf{\Theta} \times \mathcal{I} \to \mathbb{R}$,

Find: $\theta^* \in \arg\min_{\theta \in \mathbf{\Theta}} \mathbb{E}_{\pi \sim \mathcal{D}}(m(\theta, \pi))$.

Configuration of a SAT Solver for Verification [Hutter et al, 2007]

Formal verification

Software verification [Babić & Hu; CAV '07]

Hardware verification (Bounded model checking) [Zarpas; SAT '05]

Recent progress based on SAT solvers

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CDCL solver for SAT-based verification

- SPEAR, developed by Domagoj Babić at UBC
- 26 parameters, 8.34×10^{17} configurations

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Compared to manually-engineered default
- 1 week of performance tuning
- Competitive with the state of the art
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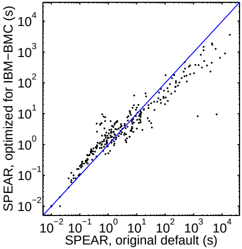

4.5-fold speedup
on hardware verification

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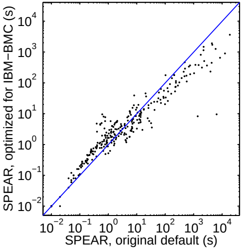

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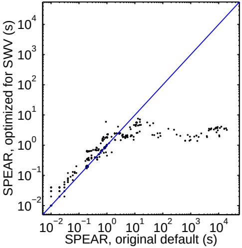

500-fold speedup $\rightsquigarrow$ won category QF_BV in 2007 SMT competition

Algorithm Selection

Definition: algorithm selection

Given

a set $\mathcal{I}$ of problem instances,

a portfolio of algorithms $\mathcal{P}$,

and a cost metric $m : \mathcal{P} \times \mathcal{I} \to \mathbb{R}$,

the per-instance algorithm selection problem is to find a mapping $s : \mathcal{I} \to \mathcal{P}$ that optimizes $\sum_{\pi \in \mathcal{I}} m(s(\pi), \pi)$, the sum of cost measures achieved by running the selected algorithm $s(\pi)$ for instance π.

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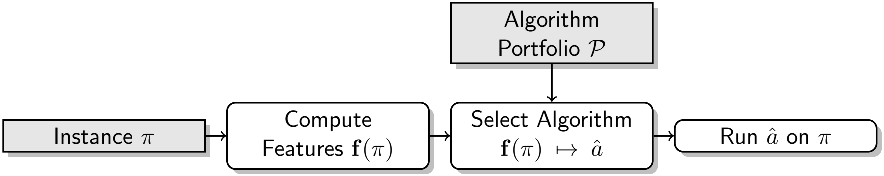

Example SAT Challenge 2012

Rank	RiG	Solver	#solved
–	–	Virtual Best Solver (VBS)	568
1	1	SATzilla2012 APP	531
2	2	SATzilla2012 ALL	515
3	1	Industrial SAT Solver	499
–	–	lingeling (SAT Competition 2011 Bronze)	488
4	2	interactSAT	480
5	1	glucose	475
6	2	SINN	472
7	3	ZENN	468
8	4	Lingeling	467
9	5	linge_dyphase	458
10	6	simpsat	453

The VBS is the best possible performance
of an algorithm selection system.
(pink: algorithm selectors, blue: portfolios, green: single-engine solvers)

Automated construction of portfolios from a single algorithm

Algorithm Configuration

- Strength: find a single configuration with strong performance for a given cost metric
- Weakness: for heterogeneous instance sets, there is often no configuration that performs great for all instances

Automated construction of portfolios from a single algorithm

- Algorithm Configuration
  - Strength: find a single configuration with strong performance for a given cost metric
  - Weakness: for heterogeneous instance sets, there is often no configuration that performs great for all instances
- Algorithm Selection
  - Strength: for heterogeneous instance sets, pick the right algorithm from a set
  - Weakness: the set to choose from typically only contains a few algorithms

Automated construction of portfolios from a single algorithm

Algorithm Configuration
- Strength: find a single configuration with strong performance for a given cost metric
- Weakness: for heterogeneous instance sets, there is often no configuration that performs great for all instances

Algorithm Selection
- Strength: for heterogeneous instance sets, pick the right algorithm from a set
- Weakness: the set to choose from typically only contains a few algorithms

Putting the two together
- Use algorithm configuration to determine useful configurations
- Use algorithm selection to select from them based on instance characteristics

Automated construction of portfolios from a single algorithm: Hydra [Xu et al. 2010, 2011]

Idea

Iteratively add configurations to a portfolio $\mathcal{P}$, starting with $\mathcal{P} = \emptyset$

In each iteration, determine configuration that is complementary to $\mathcal{P}$

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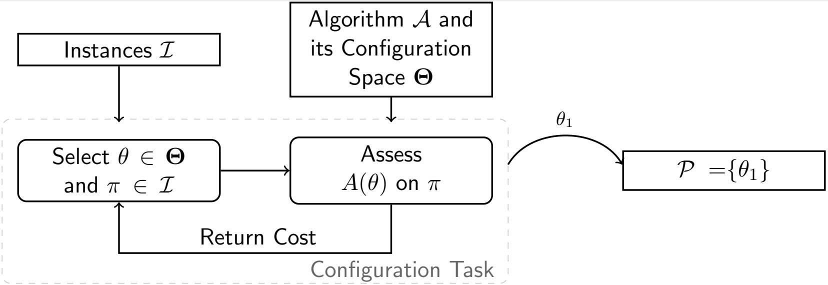

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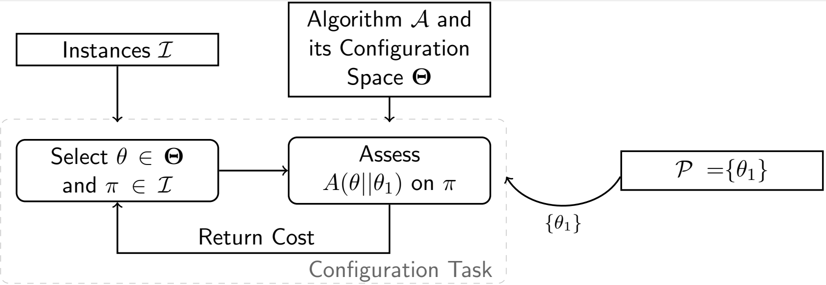

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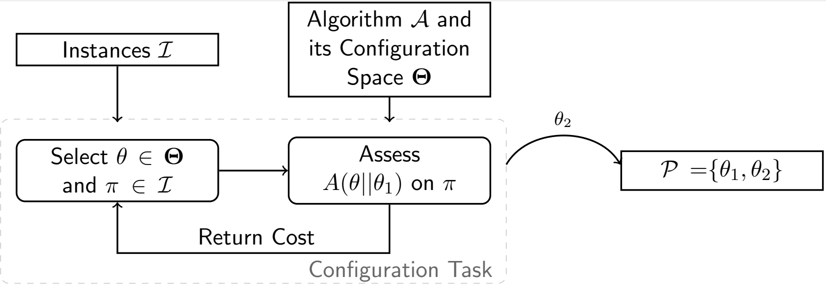

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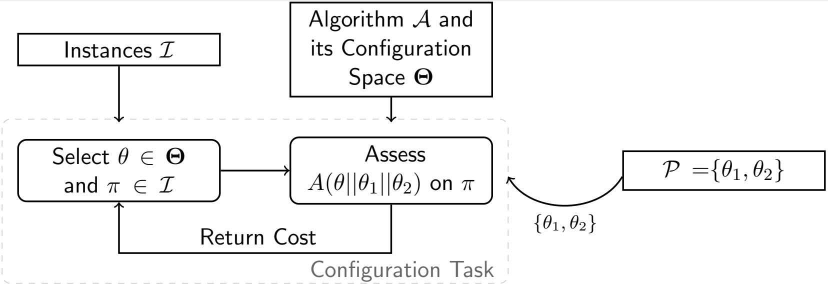


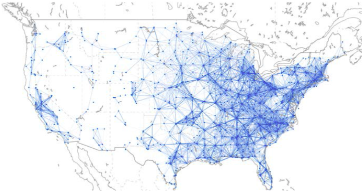
A Large-Scale Application of SAT Technology

FCC Spectrum Auction

- Wireless frequency spectra: demand increases
- US Federal Communications Commission (FCC) held 13-month auction

Key Computational Problem: feasibility testing based on interference constraints

- a hard graph colouring problem
- 2991 stations (nodes) &
- 2.7 million interference constraints
- Need to solve many different instances
- More instances solved: higher revenue



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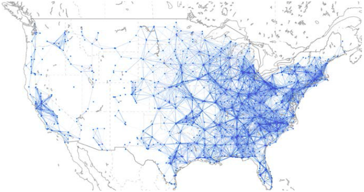

Best solution: based on SAT solving & meta-algorithmic improvements

CDCL Solver Clasp, optimized with algo. configuration method SMAC

Instance-specific configuration with Hydra (using SATzilla for algo. selection)

Concluding Remarks

- DPLL: combines simplification, unit-propagation and backtracking
  - Very efficient implementation techniques
  - Good branching heuristics
  - Clause learning
- Incomplete randomized SAT-solvers
  - Perform best on random satisfiable problem instances
- State of the art
  - Typically obtained by automatic algorithm configuration & selection