Exercise 1: Distance-Only Sensor

In this exercise, you try to locate your friend using her cell phone signals. Suppose that in the map of Freiburg, the campus of the University of Freiburg is located at \( m_0 = (10, 8)^T \), and your friend’s home is situated at \( m_1 = (6, 3)^T \). You have access to the data received by two cell towers, which are located at the positions \( x_0 = (12, 4)^T \) and \( x_1 = (5, 7)^T \), respectively. The distance between your friend’s cell phone and the towers can be computed from the intensities of your friend’s cell phone signals. These distance measurements are disturbed by independent zero-mean Gaussian noise with variances \( \sigma_0^2 = 1 \) for tower 0 and \( \sigma_1^2 = 1.5 \) for tower 1. You receive the distance measurements \( d_0 = 3.9 \) and \( d_1 = 4.5 \) from the two towers.

(a) Is your friend more likely to be at home or at the university? Explain your calculations.

(b) Implement a function in Python which generates a 3D-plot of the likelihood \( p(z|m) \) over all locations \( m \) in the vicinity of the towers. Furthermore, mark \( m_0, m_1, x_0 \) and \( x_1 \) in the plot. Is the likelihood function which you plotted a probability density function? Give a reason for your answer.

(c) Now, suppose you have prior knowledge about your friend’s habits which suggests that your friend currently is at home with probability \( P(\text{at home}) = 0.7 \), at the university with \( P(\text{at university}) \) \( = 0.3 \), and at any other place with \( P(\text{other}) = 0 \). Use this prior knowledge to recalculate the likelihoods of a).

Exercise 2: Sensor Model

Remark: This exercise is to be solved without Python.

Assume you have a robot equipped with a sensor capable of measuring the distance and bearing to landmarks. The sensor furthermore provides you with the identity of the observed landmarks.
A sensor measurement  \( z = (z_r, z_\theta)^T \) is composed of the measured distance  \( z_r \) and the measured bearing  \( z_\theta \) to the landmark  \( l \). Both the range and the bearing measurements are subject to zero-mean Gaussian noise with variances  \( \sigma_r^2 \) and  \( \sigma_\theta^2 \), respectively. The range and the bearing measurements are independent of each other.

A sensor model

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p(z \mid x, l)
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models the probability of a measurement  \( z \) of landmark  \( l \) observed by the robot from pose  \( x \).

Design a sensor model  \( p(z \mid x, l) \) for this type of sensor. Furthermore, explain your sensor model.