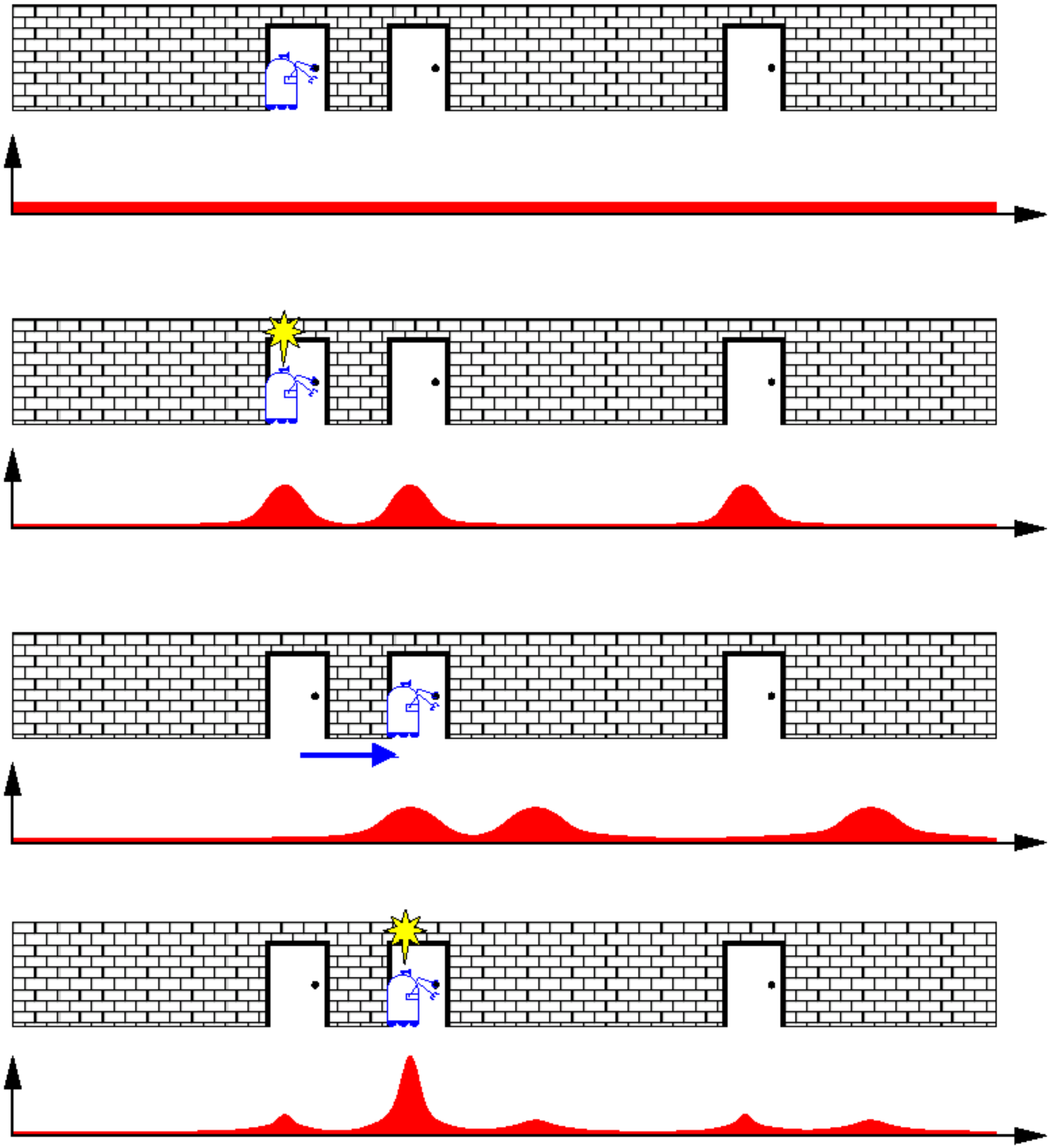
Introduction to Mobile Robotics

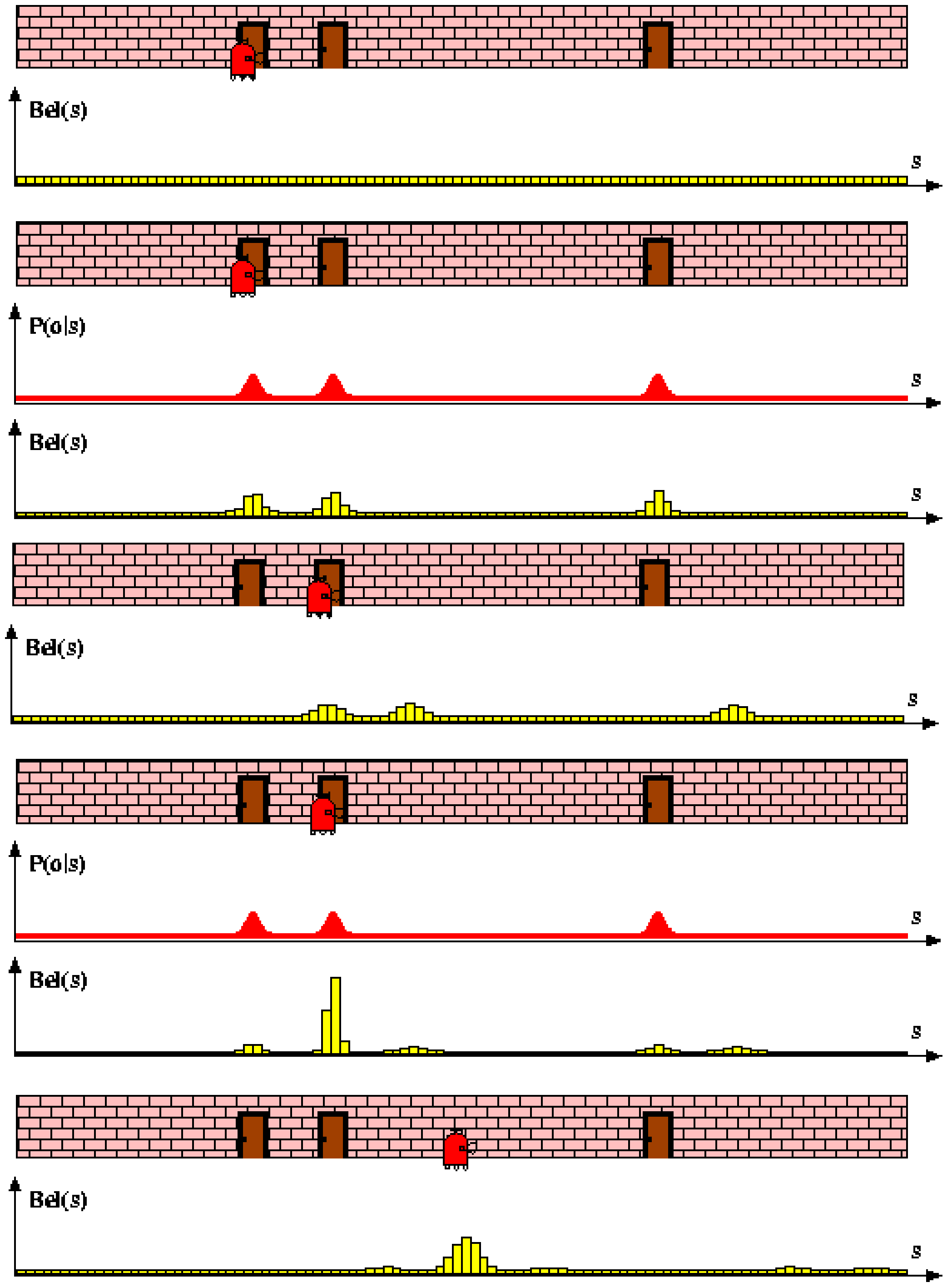
Bayes Filter – Discrete Filters

Wolfram Burgard

$$Bel(x \mid z, u) \;=\; \alpha\, p(z \mid x) \int_{x'} p(x \mid u, x')\; Bel(x')dx'$$



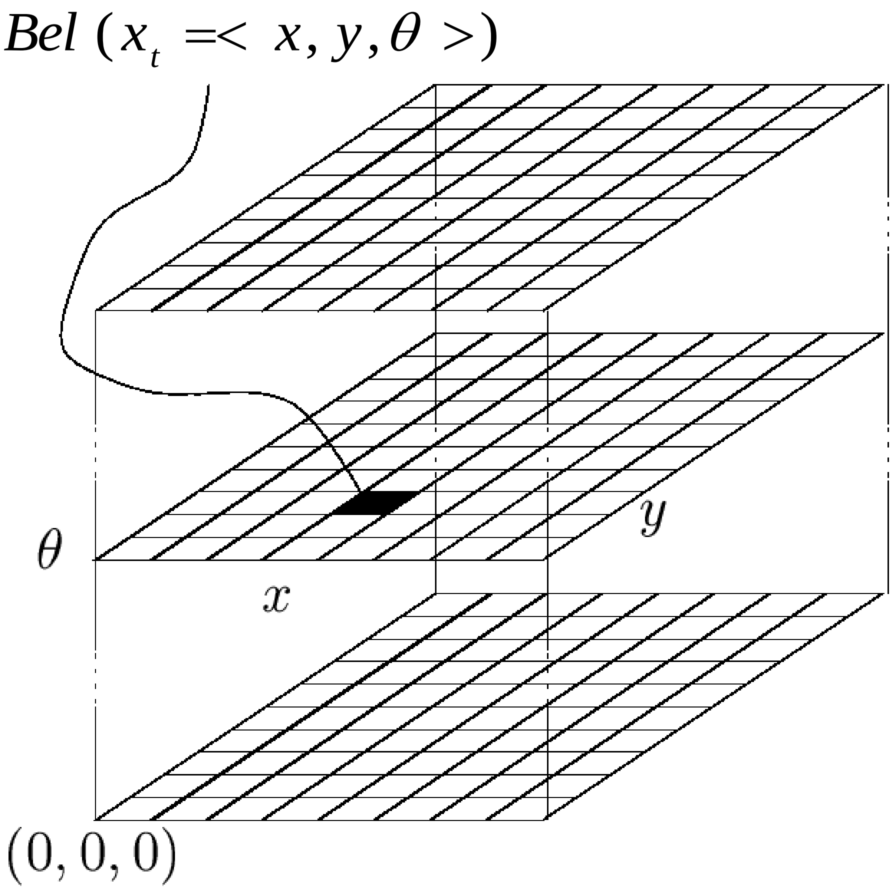
Piecewise Constant



Discrete Bayes Filter Algorithm

1. Algorithm **Discrete_Bayes_filter**(*Bel(x),d*):
2. $\eta=0$
3. If *d* is a perceptual data item *z* then
4. For all *x* do
5. $$Bel'(x) = P(z \mid x) Bel(x)$$
6. $$\eta = \eta + Bel'(x)$$
7. For all *x* do
8. $$Bel'(x) = \eta^{-1} Bel'(x)$$
9. Else if *d* is an action data item *u* then
10. For all *x* do
11. $$Bel'(x) = \sum_{x'} P(x \mid u, x')\, Bel(x')$$
12. Return *Bel'(x)*

Piecewise Constant Representation



Implementation (1)

- To update the belief upon sensory input and to carry out the normalization one has to iterate over all cells of the grid.
- Especially when the belief is peaked (which is generally the case during position tracking), one wants to avoid updating irrelevant aspects of the state space.
- One approach is not to update entire sub-spaces of the state space.
- This, however, requires to monitor whether the robot is de-localized or not.
- To achieve this, one can consider the likelihood of the observations given the active components of the state space.

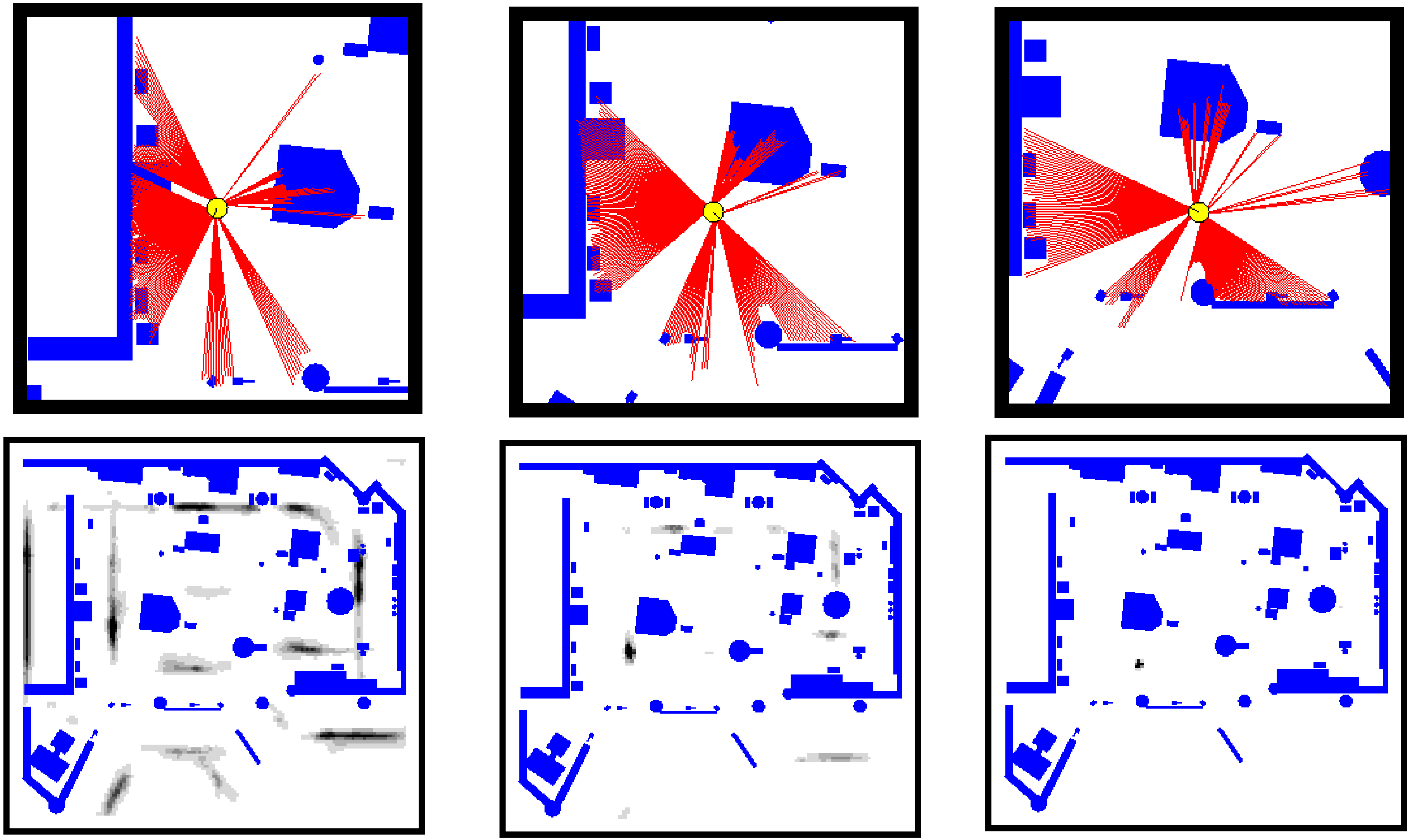
Implementation (2)

- To efficiently update the belief upon robot motions, one typically assumes a bounded Gaussian model for the motion uncertainty.
- This reduces the update cost from $O(n^2)$ to $O(n)$, where n is the number of states.
- The update can also be realized by shifting the data in the grid according to the measured motion.
- In a second step, the grid is then convolved using a separable Gaussian Kernel.
- Two-dimensional example:

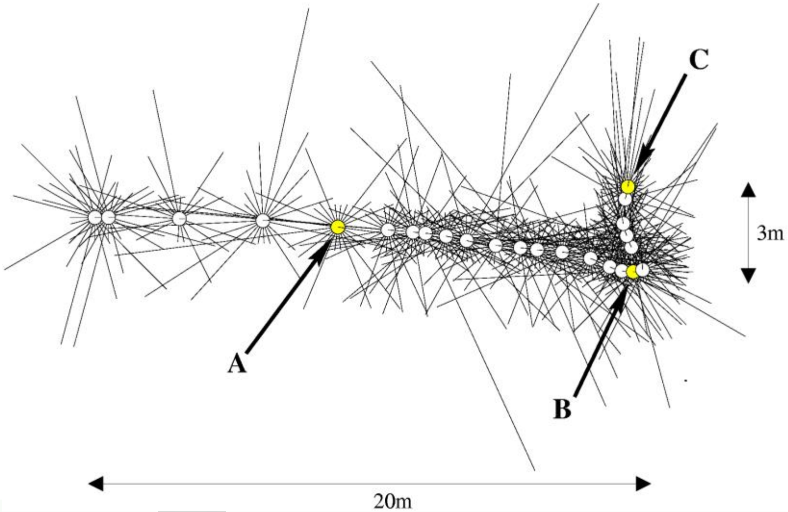
$$\begin{array}{|c|c|c|}\hline 1/16 & 1/8 & 1/16 \\ \hline 1/8 & 1/4 & 1/8 \\ \hline 1/16 & 1/8 & 1/16 \\ \hline \end{array} \;\cong\; \begin{array}{|c|}\hline 1/4 \\ \hline 1/2 \\ \hline 1/4 \\ \hline \end{array} \;+\; \begin{array}{|c|c|c|}\hline 1/4 & 1/2 & 1/4 \\ \hline \end{array}$$

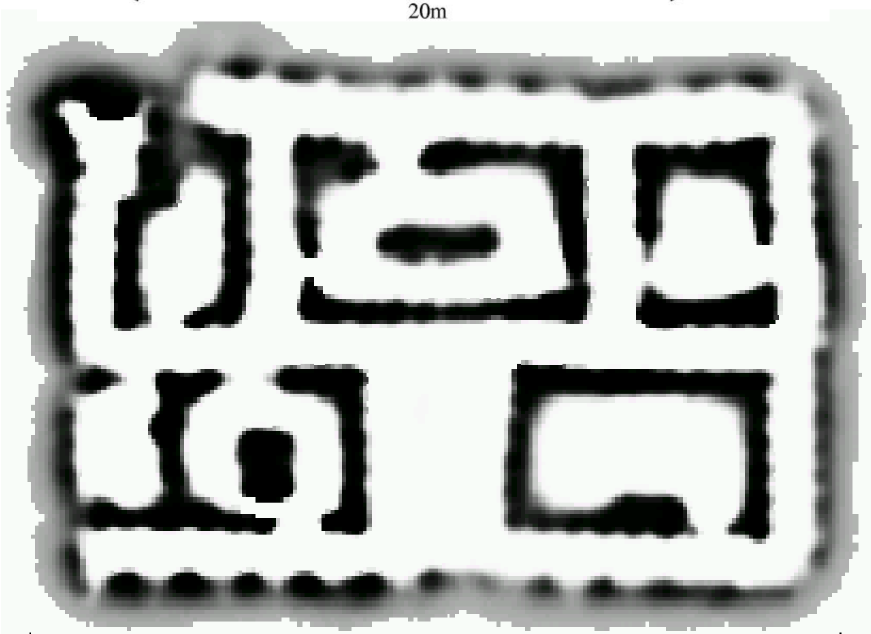
  - Fewer arithmetic operations
  - Easier to implement

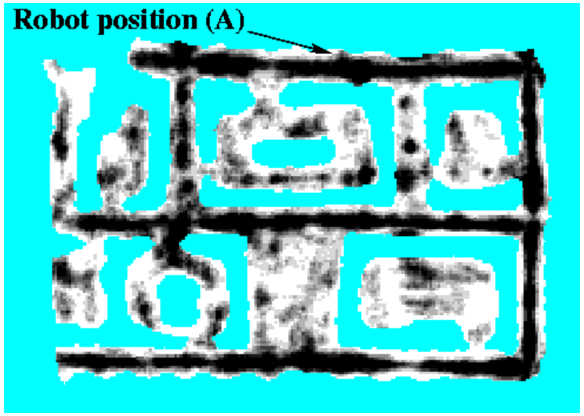
Grid-based Localization

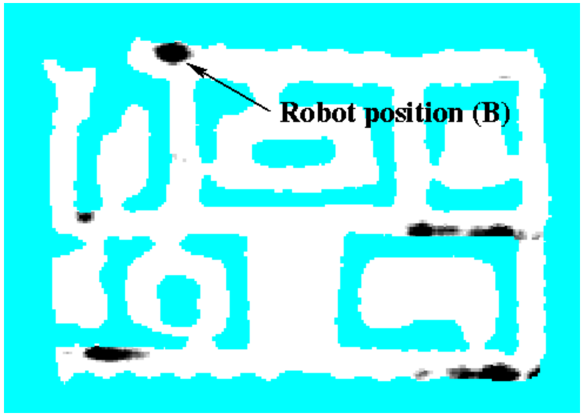


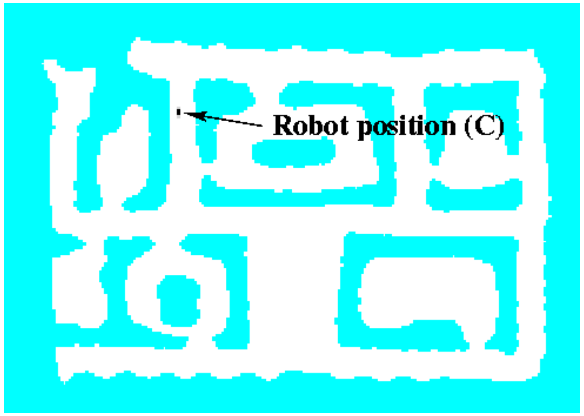
Sonars and Occupancy Grid Map





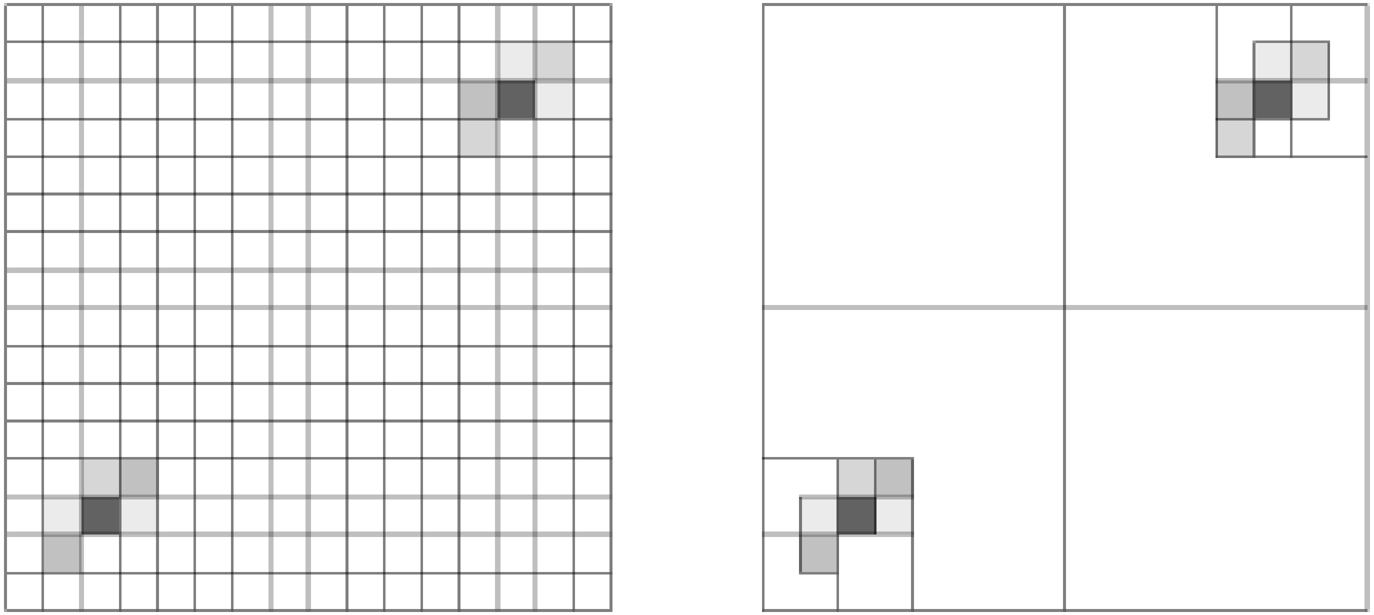






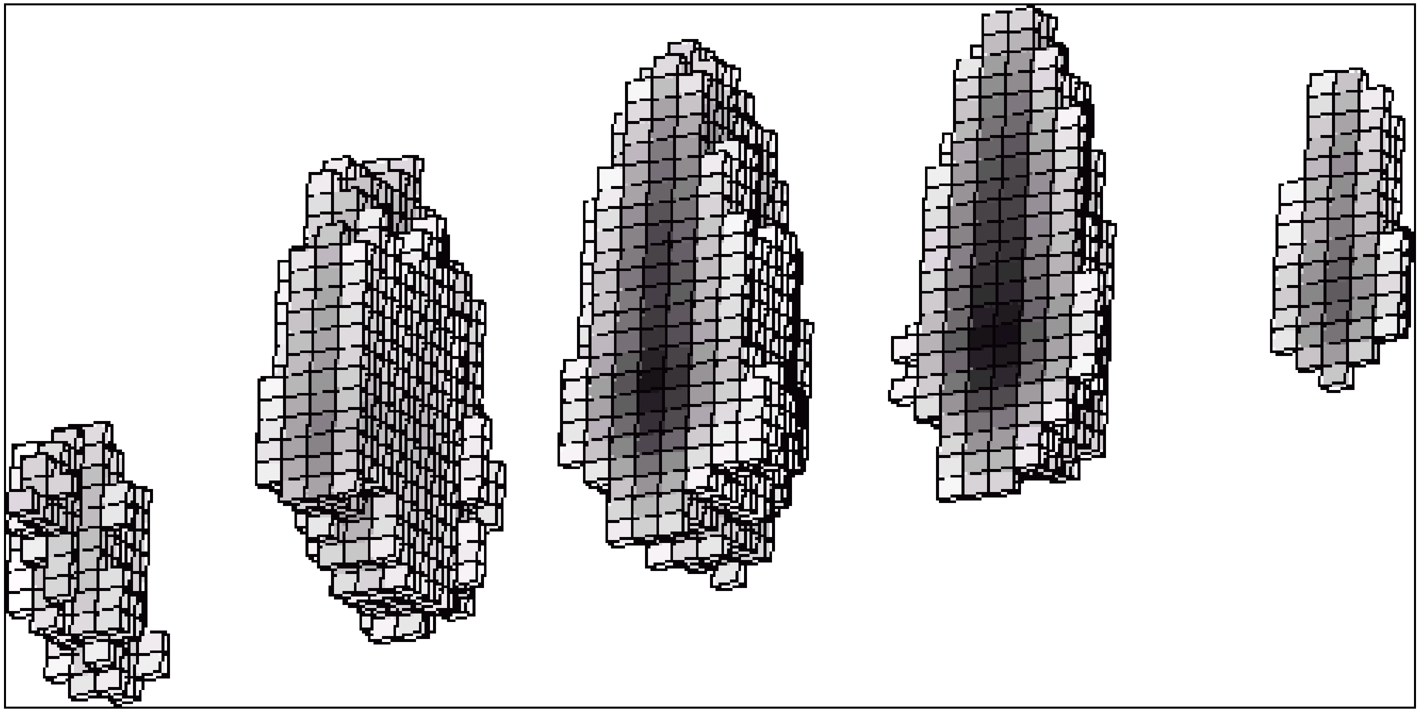
Tree-based Representation

Idea: Represent density using a variant of octrees



Tree-based Representations

- Efficient in space and time
- Multi-resolution



Summary

- Discrete filters are an alternative way for implementing Bayes Filters
- They are based on histograms for representing the density.
- They have huge memory and processing requirements
- Can easily recover from localization errors
- Their accuracy depends on the resolution of the grid.
- Special approximations need to be made to make this approach having dynamic memory and computational requirements.