Introduction to Mobile Robotics

Wheeled Locomotion

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Locomotion of Wheeled Robots

Locomotion (Oxford Dict.): Power of motion from place to place

- Differential drive (lawn mover, cleaning robots)
- Ackerman drive (cars)
- Synchronous drive
- XR4000
- Mecanum wheels

Wheels rotate around the x axis and possibly z axis
Instantaneous Center of Curvature

- For rolling motion: axis need to meet in one point
Differential Drive

\[ ICC = [x - R \sin \theta, y + R \cos \theta] \]

\[ R\omega = v \]
\[ \omega(R + l / 2) = v_r \]
\[ \omega(R - l / 2) = v_l \]
\[ R = \frac{l (v_l + v_r)}{2 (v_r - v_l)} \]
\[ \omega = \frac{v_r - v_l}{l} \]
\[ v = \frac{v_r + v_l}{2} \]
**Differential Drive: Forward Kinematics**

\[
\begin{bmatrix}
x' \\
y' \\
\theta'
\end{bmatrix} =
\begin{bmatrix}
\cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\
\sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x - ICC_x \\
y - ICC_y \\
\theta
\end{bmatrix}
+ \begin{bmatrix}
ICC_x \\
ICC_y \\
\omega \delta t
\end{bmatrix}
\]

\[
x(t) = \int_{0}^{t} v(t') \cos[\theta(t')] dt'
\]

\[
y(t) = \int_{0}^{t} v(t') \sin[\theta(t')] dt'
\]

\[
\theta(t) = \int_{0}^{t} \omega(t') dt'
\]
Differential Drive: Forward Kinematics

\[
\begin{bmatrix}
x' \\
y' \\
\theta'
\end{bmatrix} = \begin{bmatrix}
\cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\
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\end{bmatrix} \begin{bmatrix}
x - ICC_x \\
y - ICC_y \\
\theta
\end{bmatrix} + \begin{bmatrix}
ICC_x \\
ICC_y \\
\omega \delta t
\end{bmatrix}
\]

\[
x(t) = \frac{1}{2} \int_{0}^{t} [v_r(t') + v_l(t')] \cos[\theta(t')] dt'
\]

\[
y(t) = \frac{1}{2} \int_{0}^{t} [v_r(t') + v_l(t')] \sin[\theta(t')] dt'
\]

\[
\theta(t) = \frac{1}{l} \int_{0}^{t} [v_r(t') - v_l(t')] dt'
\]
Acknowledgment Drive

\[ ICC = [x - R \sin \theta, y + R \cos \theta] \]

\[ R = \frac{d}{\tan \varphi} \]

\[ \omega(R + l/2) = v_r \]

\[ \omega(R - l/2) = v_l \]

\[ R = \frac{l (v_l + v_r)}{2 (v_r - v_l)} \]

\[ \omega = \frac{v_r - v_l}{l} \]
Synchronous Drive

\[ v(t) \]

\[ \omega(t) \]
XR4000 Drive
Mecanum Wheels

\[
\begin{align*}
\nu_y &= \left( \nu_0 + \nu_1 + \nu_2 + \nu_3 \right) / 4 \\
\nu_x &= \left( \nu_0 - \nu_1 + \nu_2 - \nu_3 \right) / 4 \\
\nu_\theta &= \left( \nu_0 + \nu_1 - \nu_2 - \nu_3 \right) / 4 \\
\nu_{error} &= \left( \nu_0 - \nu_1 - \nu_2 + \nu_3 \right) / 4
\end{align*}
\]
Example: KUKA youBot
Tracked Vehicles
Other Robots: OmniTread

[courtesy by Johann Borenstein]
Other Robots: Humanoids
Holonomic and Non-Holonomic Constraints

- Holonomic: reduced configuration space
  - E.g., a train on tracks: not all positions and orientations on the plane are possible

- Non-holonomic: reduced control space
  - Limits of the possible incremental movements within the configuration space of the robot
  - E.g., a robot on a plane is not able to move sideways
Drives with Non-Holonomic Constraints

Limited to circular trajectories:
- Differential drive
- Ackermann drive
- Synchro-drive
Drives without Non-Holonomic Constraints

- Mecanum wheels
Dead Reckoning and Odometry

- Estimating the motion based on the issued controls/wheel encoder readings
- Integrated over time
Summary

- Introduced different types of drives for wheeled robots
- Math to describe the motion of the basic drives given the speed of the wheels
- Non-holonomic constraints
- Odometry and dead reckoning