$$\vec{e}_i(x) = z_i - f_i(x) \qquad e_i(x) = \vec{e}_i^{\,T} \Omega \vec{e}_i$$

$$F(x) = \sum_i e_i(x)$$

Goal : $x^* = \underset{x}{\arg\min}\, F(x) = \underset{x}{\arg\min} \sum \vec{e}_i^{\,T} \Omega\, \vec{e}_i$

1) Approximate $\vec{e}_i(x)$ by a linear function around x

$$\vec{e}_i(x+\Delta x) \approx \vec{e}_i(x) + J_i \Delta\vec{x}$$

$$\hookrightarrow J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix}$$

$\Rightarrow e_i(x) = \vec{e}_i^{\,T} \Omega\, \vec{e}_i$

$\Rightarrow$

$$\begin{aligned}
e_i(x+\Delta x) &= \vec{e}_i^{\,T}(\vec{x}+\Delta\vec{x})\, \Omega\, \vec{e}_i(\vec{x}+\Delta\vec{x}) \\
&\approx (\vec{e}_i(\vec{x}) + J_i \Delta\vec{x})^T\, \Omega\, (\vec{e}_i(\vec{x}) + J_i \Delta\vec{x}) \\
&= \vec{e}_i^{\,T} \Omega\, \vec{e}_i + \vec{e}_i^{\,T} \Omega J_i \Delta\vec{x} + \underbrace{\Delta\vec{x}^T J_i^T \Omega\, \vec{e}_i}_{=(\vec{e}_i^{\,T} \Omega J_i \Delta\vec{x})^T} \\
&\quad + \Delta\vec{x}^T J_i^T \Omega J_i \Delta x \\
&= \underbrace{\vec{e}_i^{\,T} \Omega\, \vec{e}_i}_{=c_i} + 2 \underbrace{\vec{e}_i^{\,T} \Omega J_i}_{=\vec{b}_i^T} \Delta\vec{x} + \Delta\vec{x}^T \underbrace{J_i^T \Omega J_i}_{=H_i} \Delta\vec{x} \\
&= c_i + 2\vec{b}_i^T \Delta\vec{x} + \Delta\vec{x}^T H_i \Delta\vec{x}
\end{aligned}$$

Write the global squared error function F in this way

$$\begin{aligned}
F(\vec{x}+\Delta\vec{x}) &\approx \sum_i (c_i + 2\vec{b}_i^T \Delta\vec{x} + \Delta\vec{x}^T H_i \Delta\vec{x}) \\
&= \underbrace{\sum_i c_i}_{=c_i} + 2\underbrace{(\sum_i \vec{b}_i^T)}_{=b^T} \Delta\vec{x} + \Delta\vec{x}^T \underbrace{(\sum_i H_i)}_{=H} \Delta\vec{x} \\
&= c + 2\, b^T \Delta\vec{x} + \Delta\vec{x}^T H \Delta\vec{x}
\end{aligned}$$

$\Rightarrow$ The global squared error function is a quadratic form

$$F(x+\Delta x) \approx c + 2\vec{b}^T \vec{\Delta x} + \vec{\Delta x}^T H \vec{\Delta x}$$

2) Derive wrt Δx

$$\frac{\partial F(x+\Delta x)}{\partial \Delta x} \approx 2\vec{b} + 2H\vec{\Delta x}$$

3) Set derivative to zero

$$0 = 2\vec{b} + 2H\vec{\Delta x}$$

$$\Rightarrow H\vec{\Delta x} = -\vec{b}$$ linear system

4) solve linear system

$$-H^{-1}b = \vec{\Delta x}^*$$

5) Update state

$$\vec{x} \leftarrow \vec{x} + \vec{\Delta x}^*$$

6) iterate