Advanced Techniques for Mobile Robotics

Odometry Calibration by Least Squares (in Octave)

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Least Squares Minimization

Repeatedly perform the following steps:

- Linearize the system around the current guess ***x*** and compute for each measurement

$$\mathbf{e}_i(\mathbf{x} + \mathbf{\Delta x}) \simeq \mathbf{e}_i(\mathbf{x}) + \mathbf{J}_i \mathbf{\Delta x} \qquad \mathbf{J}_i = \left.\frac{\partial \mathbf{e}_i(\mathbf{x} + \mathbf{\Delta x})}{\partial \mathbf{\Delta x}}\right|_{\mathbf{\Delta x}=0}$$

- Compute the terms for the linear system

$$\mathbf{b}^T = \sum_i \mathbf{e}_i^T \mathbf{\Omega}_i \mathbf{J}_i \qquad \mathbf{H} = \sum_i \mathbf{J}_i^T \mathbf{\Omega}_i \mathbf{J}_i \qquad \mathbf{e}_i = \mathbf{e}_i(\mathbf{x})$$

- Solve the system to get a new increment

$$\mathbf{\Delta x}^* \;=\; -\mathbf{H}^{-1}\mathbf{b}$$

- Updating the previous estimate

$$\mathbf{x} \leftarrow \mathbf{x} + \mathbf{\Delta x}^*$$

Odometry Calibration

- We have a robot which moves in an environment, gathering the odometry measurements $\boldsymbol{u_i}$,affected by a systematic error.
- For each $\boldsymbol{u_i}$ we have a ground truth $\boldsymbol{u^*_i}$
- There is a function $\boldsymbol{f_i(x)}$ which, given some bias parameters $\boldsymbol{x}$, returns a an unbiased odometry for the reading $\boldsymbol{u_i'}$ as follows

$$\mathbf{u}'_i = f_i(\mathbf{x}) = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} \mathbf{u}_i$$

Odometry Calibration (cont.)

- The state vector is

$$\mathbf{x} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{21} & x_{22} & x_{23} & x_{31} & x_{32} & x_{33} \end{pmatrix}^T$$

- The error function is

$$\mathbf{e}_i(\mathbf{x}) = \mathbf{u}_i^* - \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} \mathbf{u}_i$$

- Its derivative is:

$$\mathbf{J}_i = \frac{\partial \mathbf{e}_i(\mathbf{x})}{\partial \mathbf{x}} = - \begin{pmatrix} u_{i,x} & u_{i,y} & u_{i,\theta} & & & & & & \\ & & & u_{i,x} & u_{i,y} & u_{i,\theta} & & & \\ & & & & & & u_{i,x} & u_{i,y} & u_{i,\theta} \end{pmatrix}$$

Exercise

- Write a program to calibrate the odometry
- We provide an input file obtained from a real robot.
- Format of z.dat:
  - Every line is a single odometry measurement

    $u'_x \; u'_y \; u'_t \; u_x \; u_y \; u_t$

  - **u'** and **u** are respectively the true and the measured odometry of the system in relative coordinates (e.g. motion of the robot between two consecutive frames).

Exercise (in sequential steps)

- Load the measurements (into a matrix)
- Write a function ***A***=*v2t(**u**)* that given a transformation expressed as a vector ***u***=**[**u_x u_y u_t**]** returns an homogeneous transformation matrix ***A.***
- Write a function ***u***=*t2v**(A)*** dual of the previous one.
- Write a function ***T***=*compute_odometry_trajectory**(U)*** that computes a trajectory in the global frame by chaining up the measurements (rows) of the Nx3 matrix U (to visualize the data). *Hint: use the two functions defined above. Test it on the input data by displaying the trajectories.*
- Define the error function $e_i(X)$ for a line of the measurement matrix. Call it *error_function(**i,X,Z**)*.
- Define the Jacobian function for the measurement ***i*** (call it *jacobian**(i,Z)**.*
- Write a function ***X***=*ls_calibrate_odometry**(Z)*** which constructs and solves the quadratic problem. It should return the calibration parameters ***X.***
- Write a function ***Uprime***=*apply_odometry_correction**(X,U)*** which applies the correction to all odometries in the Nx3 matrix U. Test the computed calibration matrix and generate a trajectory.
- Plot the real, the estimated and the corrected odometry.
- In the directory you will find an octave script 'LsOdomCalib' which you can use to test your program.

Transformation Functions v2t & t2v

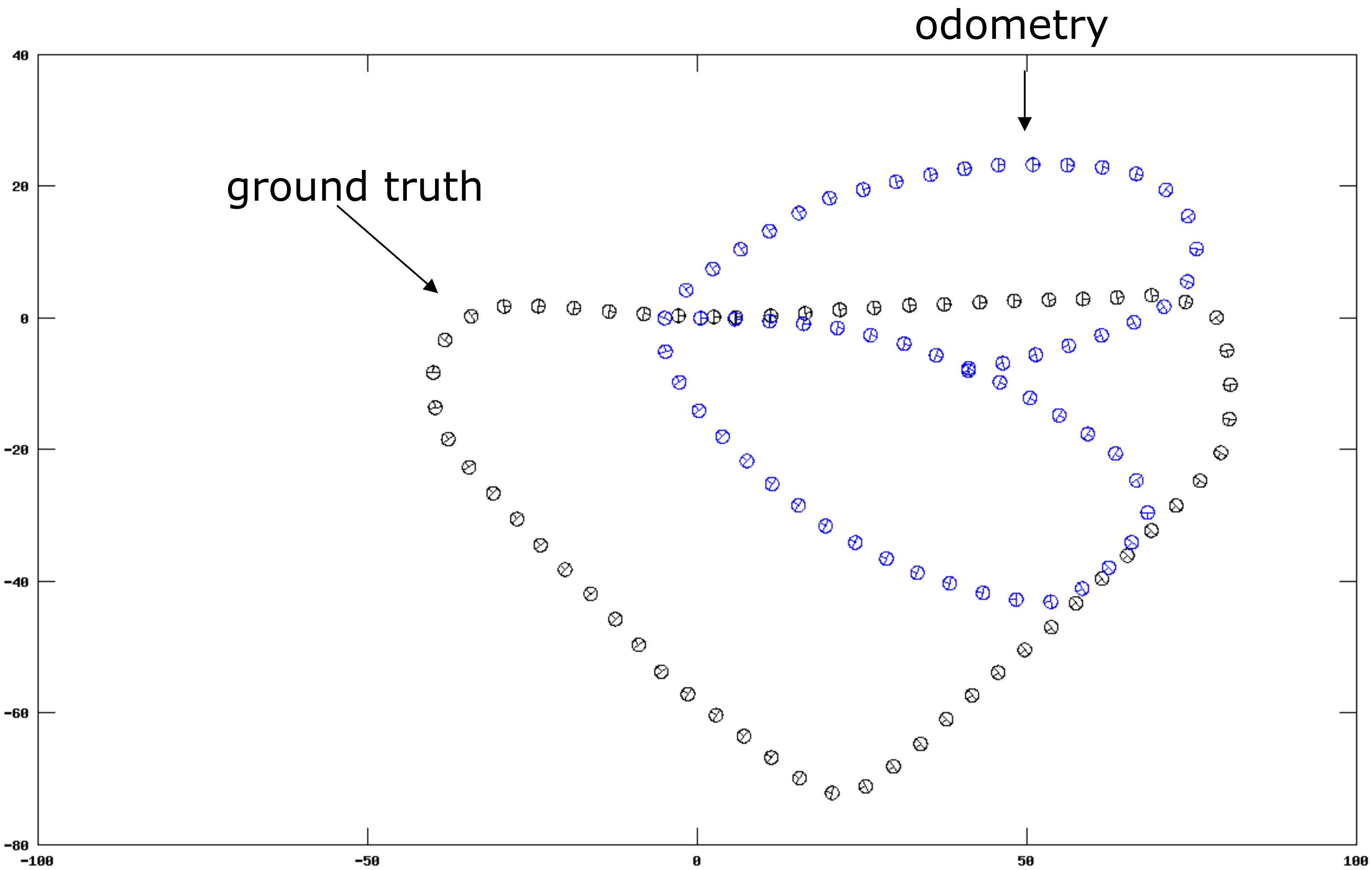
```
function v=t2v(A)
   v(1:2, 1)=A(1:2,3);
   v(3,1)=atan2(A(2,1),A(1,1));
endfunction

function A=v2t(v)
   c=cos(v(3));
   s=sin(v(3));
   A=
   [c, -s, v(1) ;
    s,  c, v(2) ;
    0   0  1   ];
endfunction
```

compute_odometry_trajectory

```
function T=compute_odometry_trajectory(U)
    T=zeros(size(U,1),3);
    P=v2t(zeros(1,3));
    for i=1:size(U,1),
        u=U(i,1:3)';
        P*=v2t(u);
        T(i,1:3)=t2v(P)';
    end
end
```

Trajectories



Error function

```
function e=error_function(i,X,Z)
    uprime=Z(i,1:3)';
    u=Z(i,4:6)';
    e=uprime-X*u;
end
```

Jacobian

```
function A=jacobian(i,Z)
   u=Z(i,4:6);
   A=zeros(3,9);
   A(1,1:3)=-u;
   A(2,4:6)=-u;
   A(3,7:9)=-u;
end
```

Quadratic Solver

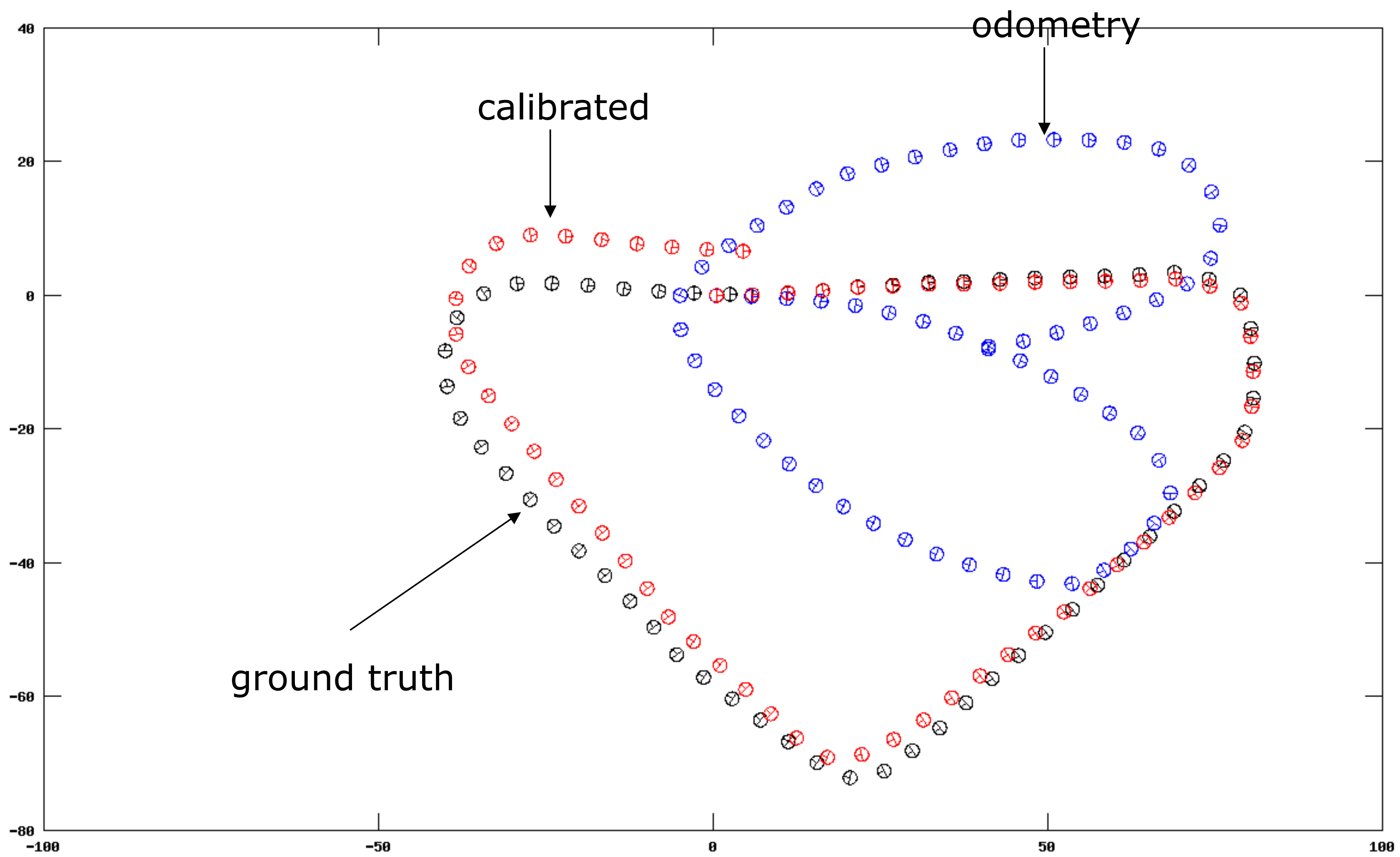
```
function X=ls_calibrate_odometry(Z)
    #accumulator variables for the linear system
    H=zeros(9,9);
    b=zeros(9,1);
    #initial solution (can be anything, se set it to the identity transformation)
    X=eye(3);

    #loop through the measurements and update the accumulators
    for i=1:size(Z,1),
        e=error_function(i,X,Z);
        A=jacobian(i,Z);
        H=H+A'*A;
        b=b+A'*e;
    end
    #solve the linear system
    deltaX=-H\b;
    #this reshapes the 9x1 increment vector in a 3x3 atrix
    dX=reshape(deltaX,3,3)';
    #computes the cumulative solution
    X=X+dX;
end
```

applyOdometryCorrection

```
function C=applyOdometryCorrection(bias, U)
    C=zeros(size(U,1),3);
    for i=1:size(U,1),
        u=U(i,1:3)';
        uc=bias*u;
        C(i,:)=uc';
    end
endfunction
```

Plots



odometry
calibrated
ground truth
40
20
0
-20
-40
-60
-80
-100
-50
0
50
100

Questions

- Which one of the wheels of the robot was deflated (right or left)?
- Do you feel confident to apply least squares to more complex problems?

For next week you should have understood the basic concepts of least squares minimization.