Advanced Techniques for Mobile Robotics

Simultaneous Calibration, Localization, and Mapping

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• Constraints connect the poses of the robot while it is moving
• Constraints are inherently uncertain
Observing previously seen areas generates constraints between non-successive poses.

Constraints are inherently uncertain.
SCLAM – Adding Calibration

- Eliminate systematic errors
  - Location of the sensor on the robot
  - Systematic odometry errors
Relevance

- Systematic errors can strongly influence the results of a mapping system
Key Idea

- Extend graph-based SLAM to estimate also systematic errors
- Explicitly model that the measurements are obtained in a different coordinate frame
- Estimate the forward kinematics parameters
- Allow for online optimization (e.g., when the robot carries a load)

sensor placement  \[\rightarrow\]  base line

wheel size
For rolling motion to occur, each wheel has to move along its y-axis.
Roboticas I
Differential Drive

ICC = [x − R sin θ, y + R cos θ]

ω(R + l / 2) = v_r
ω(R − l / 2) = v_l

R = \frac{l (v_l + v_r)}{2 (v_r − v_l)}

ω = \frac{v_r − v_l}{l}
v = \frac{v_r + v_l}{2}

=b in this lecture
Forward Kinematics

\[
\begin{bmatrix}
x' \\
y' \\
\theta'
\end{bmatrix} = \begin{bmatrix}
\cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\
\sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\
0 & 0 & 1
\end{bmatrix}\begin{bmatrix}
x - ICC_x \\
y - ICC_y \\
\theta
\end{bmatrix} + \begin{bmatrix}
ICC_x \\
ICC_y \\
\omega \delta t
\end{bmatrix}
\]
Odometry Measurements

Forward kinematics for a differential drive robot:

\[ K(u, k) = \begin{pmatrix} R(\Delta t\omega) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -ICC \\ 0 \end{pmatrix} + \begin{pmatrix} ICC \\ \Delta t\omega \end{pmatrix} \]

\[ k = \begin{pmatrix} r_l \\ r_r \\ b \end{pmatrix}, \quad u = \begin{pmatrix} v_l \\ v_r \end{pmatrix} \]

\[ \omega = \frac{v_rr_r - v_lr_l}{b} \]

Error function:

\[ e_{i}^{u}(x) = \left( x_{i+1} \ominus x_{i} \right) \ominus K(u_{i}, k) \]

robot poses \quad odometry
Sensor Measurements

The observations (★) allow to estimate the ego-motion of the sensor and thus the motion of the robot, **given** the position of the sensor on the robot.

Error function:

$$e_{ij}^{1}(x) = \left( (x_{j} \oplus 1) \ominus (x_{i} \oplus 1) \right) \ominus z_{ij}$$

- position of the sensor on the robot
- robot poses
- scan matching
Graph Optimization

- Combine both types of measurements
- Find the minimum of the error function:

\[
F(x, l, k) = \sum_{\langle i, j \rangle} e_{ij}^l(x)^\top \Omega_{ij}^z e_{ij}^l(x) + \sum_i e_i^u(x)^\top \tilde{\Omega}_i^u e_i^u(x)
\]
Graph Optimization (2)

- Without loss of generality: $y := (x, k, l)^{\top}$
- Find the minimum of the error function

Minimization by applying methods such as Gauss-Newton or Levenberg-Marquardt
Iterative Solution (1)

- Linearize the error around the current solution $y_0$ by fixing $y$ and varying a small increment $\Delta y$

$$e_k(y_0 \oplus \Delta y) = e_k + J_k \Delta y \quad J_k = \left. \frac{\partial e_k(y \oplus \Delta y)}{\partial \Delta y} \right|_{\Delta y=0}$$

- The error term in the neighborhood of the linearization becomes a quadratic form

$$e_k(y_0 \oplus \Delta y) \approx e_k^T(y_0 \oplus \Delta y) \Omega_k e_k(y_0 \oplus \Delta y) + 2 e_k^T \Omega_k J_k \Delta y + \Delta y^T J_k^T \Omega_k J_k \Delta y$$

$$= c_k + 2 b_k^T \Delta y + \Delta y^T H_k \Delta y$$
The same substitution can be applied to the global error function

\[
F(y_0 \oplus \Delta y) \simeq \sum_k \left( c_k + b_k^T \Delta y + \Delta y^T H_k \Delta y \right)
\]

\[
= \sum_k c_k + 2 \left( \sum_k b_k^T \right) \Delta y + \Delta y^T \left( \sum_k H_k \right) \Delta y
\]

\[
= c + 2b^T \Delta y + \Delta y^T H \Delta y
\]
Iterative Solution (3)

- The optimum of the quadratic form can be found by solving the linear system

\[ H \Delta y = -b \]

- or using the damped variant

\[ (H + \lambda I) \Delta y = -b \]

- The improved estimate is obtained by applying the perturbation to the previous guess

\[ y_0 \leftarrow y_0 \oplus \Delta y^* \]
How Does This Work in Practice?

- Three differential drive robots
- Equipped with laser range finders

PowerBot  Custom  Pioneer I
Effect of the Odometry Parameters

with online calibration

without online calibration
## Simulation

- Simulate a robot carrying a load
- Sliding windows for the wheel radii
Online Odometry Calibration

- Robot carries a load
- Additional weight compresses the tires
- Since the load is variable, the best performance can be obtained by estimating the parameters online

![Graph showing wheel radius and weight changes over time.]
Position of the on-board sensor

Ground truth: (0.3, 0.6, 30°)
# Offline Experiments – Real world

## Robot parameters

<table>
<thead>
<tr>
<th></th>
<th>PowerBot</th>
<th>Custom</th>
<th>Pioneer</th>
</tr>
</thead>
<tbody>
<tr>
<td>wheel radius [m]</td>
<td>0.125</td>
<td>0.16</td>
<td>0.065</td>
</tr>
<tr>
<td>wheel distance [m]</td>
<td>0.56</td>
<td>0.7</td>
<td>0.35</td>
</tr>
<tr>
<td>ticks per revolution</td>
<td>22835</td>
<td>20000</td>
<td>1970</td>
</tr>
<tr>
<td>laser offset [m, m, °]</td>
<td>(0.22, 0, 0)</td>
<td>(0.3, 0, 0)</td>
<td>(0.1, 0, 0)</td>
</tr>
<tr>
<td>laser scanner model</td>
<td>Sick LMS291</td>
<td>Sick LMS151</td>
<td>Hokuyo URG</td>
</tr>
</tbody>
</table>

## Calibration results

<table>
<thead>
<tr>
<th></th>
<th>laser offset (m, m, °)</th>
<th>wheel radii (m, m)</th>
<th>distance m</th>
</tr>
</thead>
<tbody>
<tr>
<td>PowerBot - 1</td>
<td>(0.2258, 0.0026, 0.099)</td>
<td>(0.1263, 0.1275)</td>
<td>0.5825</td>
</tr>
<tr>
<td>PowerBot - 2</td>
<td>(0.2231, -0.0031, 0.077)</td>
<td>(0.1243, 0.1248)</td>
<td>0.6091</td>
</tr>
<tr>
<td>Custom - 1</td>
<td>(0.3067, -0.0051, -0.357)</td>
<td>(0.1603, 0.1605)</td>
<td>0.6969</td>
</tr>
<tr>
<td>Custom - 2</td>
<td>(0.3023, -0.0087, -0.013)</td>
<td>(0.1584, 0.1575)</td>
<td>0.7109</td>
</tr>
<tr>
<td>Pioneer - 1</td>
<td>(0.1045, 0.009, -0.178)</td>
<td>(0.0656, 0.065)</td>
<td>0.3519</td>
</tr>
<tr>
<td>Pioneer - 2</td>
<td>(0.1066, -0.0031, -0.28)</td>
<td>(0.0658, 0.0655)</td>
<td>0.3461</td>
</tr>
</tbody>
</table>
Influence of the Underground

- Provides additional information about the noise induced by the floor
Summary

- Additional Parameters can be estimated during mapping
- Here: sensor offset and odometry parameters
- Parameter estimation can be easily integrated into the error minimization framework