Exercise: Graph-Based SLAM

Implement a least-squares method to address SLAM in its graph-based formulation. To support this task, we provide a small Octave framework (see course website). The framework contains the following folders:

**data** contains several datasets, each gives the measurements of one SLAM problem

**octave** contains the Octave framework with stubs to complete.

**plots** this folder is used to store images.

The below mentioned tasks should be implemented inside the framework in the directory **octave** by completing the stubs:

- Implement the function in **compute_global_error.m** for computing the current error.
- Implement the function in **linearize_pose_pose_constraint.m** for computing the error and the Jacobian of a pose-pose constraint.
- Implement the function in **linearize_pose_landmark_constraint.m** for computing the error and the Jacobian of a pose-landmark constraint.
- Implement the function in **linearize_and_solve.m** for constructing and solving the linear approximation.
- Implement the update of the state vector and the stopping criterion in **lsSLAM.m**. A possible choice for the stopping criterion is \( \| \Delta x \|_\infty < \epsilon \), i.e., the maximum absolute \( \Delta x \) does not exceed \( \epsilon \).

After implementing the missing parts, you can run the framework. To do that, change into the directory octave and launch Octave. To start the main loop, type **lsSLAM**. The script will produce a plot showing the positions of the robot and (if available) the positions of the landmarks in each iteration. These plots will be saved in the **plots** directory.
The file \langle name of the dataset \rangle .png depicts the result that you should obtain after convergence for each dataset. Additionally, the initial and the final error for each dataset should be approximately:

<table>
<thead>
<tr>
<th>dataset</th>
<th>initial error</th>
<th>final error</th>
</tr>
</thead>
<tbody>
<tr>
<td>simulation-pose-pose.dat</td>
<td>138862234</td>
<td>8269</td>
</tr>
<tr>
<td>simulation-pose-landmark.dat</td>
<td>3030</td>
<td>474</td>
</tr>
<tr>
<td>intel.dat</td>
<td>1795139</td>
<td>360</td>
</tr>
<tr>
<td>dlr.dat</td>
<td>3696555336</td>
<td>56860</td>
</tr>
</tbody>
</table>

The state vector contains the following entities:

- pose of the robot: \( \mathbf{x}_i = (x_i, y_i, \theta_i)^T \)
  
  Hint: You may use the function \( \mathbf{v}_2 \mathbf{t}(\cdot) \) and \( \mathbf{t}_2 \mathbf{v}(\cdot) \):

  \[
  \mathbf{v}_2 \mathbf{t}(\mathbf{x}_i) = \begin{pmatrix} R_i \mathbf{t}_i \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix} = X_i
  \]

  \[
  \mathbf{t}_2 \mathbf{v}(X_i) = \mathbf{x}_i
  \]

- position of a landmark: \( \mathbf{x}_l = (x_l, y_l)^T \)

We consider the following error functions:

- pose-pose constraint: \( \mathbf{e}_{ij} = \mathbf{t}_2 \mathbf{v}(Z_{ij}^{-1}(X_i^{-1}X_j)) \), where \( Z_{ij} = \mathbf{v}_2 \mathbf{t}(\mathbf{z}_{ij}) \) is the transformation matrix of the measurement \( \mathbf{z}_{ij} = (t_{ij}, \theta_{ij}) \).
  
  Hint: For computing the Jacobian, write the error function with rotation matrices and translation vectors:

  \[
  \mathbf{e}_{ij} = \begin{pmatrix} R_{ij}^T(R_{ij}^T(\mathbf{t}_j - \mathbf{t}_i) - \mathbf{t}_{ij}) \\ \theta_j - \theta_i - \theta_{ij} \end{pmatrix}
  \]

- pose-landmark constraint: \( \mathbf{e}_{il} = R_{il}^T(\mathbf{x}_l - \mathbf{t}_i) - \mathbf{z}_{il} \)

Some implementation tips:

- The functions \( \mathbf{v}_2 \mathbf{t}(\cdot) \) and \( \mathbf{t}_2 \mathbf{v}(\cdot) \) to convert between a pose vector and a transformation matrix are available in the Octave framework.

- You may first implement the functions in `linearize_pose_pose_constraint.m` and apply the framework to the datasets which do not contain landmarks.

- When solving the linear system exploit the sparseness of the matrix. You may resort to the backslash operator.

- Many of the functions in Octave can handle matrices and compute values along the rows or columns of a matrix. Some useful functions that support this are \texttt{max}, \texttt{abs}, \texttt{sum}, \texttt{log}, \texttt{sqrt}, \texttt{sin}, \texttt{cos}, and many others.