Computing $\Phi_t = [G_t^T]^{-1}\ \Omega_{t-1}\ G_t^{-1}$

- Goal: constant time if Ω_{t-1} is sparse

$$\begin{aligned} G_t^{-1} &= (I + F_x^T\ \Delta\ F_x)^{-1} \\ &= \begin{pmatrix} \Delta + I_3 & 0 \\ 0 & I_{2N} \end{pmatrix}^{-1} \end{aligned}$$

3x3 identity   2Nx2N identity

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holds for every block matrices where the off-diagonal blocks are zero blocks

Computing $\Phi_t = [G_t^T]^{-1}\ \Omega_{t-1}\ G_t^{-1}$

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$$\begin{aligned} G_t^{-1} &= (I + F_x^T\ \Delta\ F_x)^{-1} \\ &= \begin{pmatrix} \Delta + I_3 & 0 \\ 0 & I_{2N} \end{pmatrix}^{-1} \\ &= \begin{pmatrix} (\Delta + I_3)^{-1} & 0 \\ 0 & I_{2N} \end{pmatrix} \\ &= I_{3+2N} + \begin{pmatrix} (\Delta + I_3)^{-1} - I_3 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

Note: 3x3 matrix

Computing $\Phi_t = [G_t^T]^{-1}\ \Omega_{t-1}\ G_t^{-1}$

- Goal: constant time if Ω_{t-1} is sparse

$$\begin{aligned} G_t^{-1} &= (I + F_x^T\ \Delta\ F_x)^{-1} \\ &= \begin{pmatrix} \Delta + I_3 & 0 \\ 0 & I_{2N} \end{pmatrix}^{-1} \\ &= \begin{pmatrix} (\Delta + I_3)^{-1} & 0 \\ 0 & I_{2N} \end{pmatrix} \\ &= I_{3+2N} + \begin{pmatrix} (\Delta + I_3)^{-1} - I_3 & 0 \\ 0 & 0 \end{pmatrix} \\ &= I + \underbrace{F_x^T\ [(I + \Delta)^{-1} - I]\ F_x}_{\Psi_t} \\ &= I + \Psi_t \end{aligned}$$