Exercise: Implement an EKF SLAM System

Implement an extended Kalman filter SLAM (EKF SLAM) system. To support this task, we provide a small Octave framework (see course website). The framework contains the following folders:

data contains files representing the world definition and sensor readings.
octave contains the EKF SLAM framework with stubs to complete.
plots this folder is used to store images.

The below mentioned tasks should be implemented inside the framework in the directory octave by completing the stubs.

After implementing the missing parts, you can run the EKF SLAM system. To do that, change into the directory octave and launch Octave. Type ekf_slam to start the main loop (this may take some time). The program plots the current belief of the robot (pose and landmarks) in the directory plots. Figure 1 depicts some example images of the state of the EKF. You can use the images for debugging and to generate an animation. For example, you can use ffmpeg from inside the plots directory as follows:

```
ffmpeg -r 10 -b 500000 -i ekf_%03d.png ekf_slam.mp4
```

(a) Implement the prediction step of the EKF SLAM algorithm in the file prediction_step.m. Use the odometry motion model:

\[
\begin{pmatrix}
  x_t \\
  y_t \\
  \theta_t
\end{pmatrix} = \begin{pmatrix}
  x_{t-1} \\
  y_{t-1} \\
  \theta_{t-1}
\end{pmatrix} + \begin{pmatrix}
  \delta_{\text{trans}} \cos(\theta_{t-1} + \delta_{\text{rot}1}) \\
  \delta_{\text{trans}} \sin(\theta_{t-1} + \delta_{\text{rot}1}) \\
  \delta_{\text{rot}1} + \delta_{\text{rot}2}
\end{pmatrix}.
\]

Compute its Jacobian \( G^p_t \) to construct the full Jacobian matrix \( G_t \):

\[
G^p_t = I + \begin{pmatrix}
0 & 0 & -\delta_{\text{trans}} \sin(\theta_{t-1} + \delta_{\text{rot}1}) \\
0 & 0 & \delta_{\text{trans}} \cos(\theta_{t-1} + \delta_{\text{rot}1}) \\
0 & 0 & 0
\end{pmatrix}.
\]
For the noise in the motion model assume

\[ R^*_t = \begin{pmatrix}
0.1 & 0 & 0 \\
0 & 0.1 & 0 \\
0 & 0 & 0.01
\end{pmatrix}. \]

(b) Implement the correction step in the file `correction_step.m`. The argument `z` of this function is a struct array containing `m` landmark observations made at time step `t`. Each observation `z(i)` has an id `z(i).id`, a range `z(i).range`, and a bearing `z(i).bearing`.

Iterate over all measurements \((i = 1, \ldots, m)\) and compute the Jacobian \(H^i_t\) (see Slide 05 Page 35ff.). You should compute a block Jacobian matrix \(H_t\) by stacking the \(H^i_t\) matrices corresponding to the individual measurements. Use it to compute the Kalman gain and update the system mean and covariance after the for-loop. For the noise in the sensor model assume that \(Q_t\) is a diagonal square matrix as follows

\[ Q_t = \begin{pmatrix}
0.01 & 0 & 0 & \cdots \\
0 & 0.01 & 0 & \cdots \\
0 & 0 & 0.01 & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix} \in \mathbb{R}^{2m \times 2m}. \]

Some implementation tips:

- While debugging, run the filter only for a few steps by replacing the for-loop in `ekf_slam.m` by something along the lines of `for t = 1:50`.
- The command `repmat` allows you to replicate a given matrix in many different ways and is magnitudes faster than using for-loops.
- When converting implementations containing for-loops into a vectorized form it often helps to draw the dimensions of the data involved on a sheet of paper.
- Many of the functions in Octave can handle matrices and compute values along the rows or columns of a matrix. Some useful functions that support this are `sum`, `sqrt`, and many others.