Robot Mapping

FastSLAM – Feature-Based SLAM with Particle Filters

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Particle Filter

- Non-parametric recursive Bayes filter
- Posterior is represented by a set of weighted samples
- Can model arbitrary distributions
- Works well in low-dimensional spaces
- 3-Step procedure
 - Sampling from proposal
 - Importance Weighting
 - Resampling

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Particle Filter Algorithm

1. Sample the particles from the proposal distribution

$$x_t^{[j]} \sim \pi(x_t \mid \ldots)$$

2. Compute the importance weights

$$w_t^{[j]} = \frac{target(x_t^{[j]})}{proposal(x_t^{[j]})}$$

3. Resampling: Draw sample i with probability $w_t^{[i]}$ and repeat J times

Particle Representation

A set of weighted samples

$$\mathcal{X} = \left\{ \left\langle x^{[i]}, w^{[i]} \right\rangle \right\}_{i=1,\dots,N}$$

- Think of a sample as one hypothesis about the state
- For feature-based SLAM:

$$x = (x_{1:t}, \frac{m_{1,x}, m_{1,y}, \dots, m_{M,x}, m_{M,y}}{\text{landmarks}})^T$$

_

Dimensionality Problem

Particle filters are effective in low dimensional spaces as the likely regions of the state space need to be covered with samples.

$$x = (x_{1:t}, m_{1,x}, m_{1,y}, \dots, m_{M,x}, m_{M,y})^T$$

high-dimensional

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Can We Exploit Dependencies
Between the Different
Dimensions of the State Space?

$$x_{1:t}, m_1, \ldots, m_M$$

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If We Know the Poses of the Robot, Mapping is Easy!

$$\frac{x_{1:t}, m_1, \ldots, m_M}{}$$

Key Idea

$$\frac{x_{1:t}, m_1, \ldots, m_M}{}$$

If we use the particle set only to model the robot's path, each sample is a path hypothesis. For each sample, we can compute an individual map of landmarks.

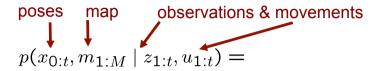
Rao-Blackwellization

 Factorization to exploit dependencies between variables:

$$p(a,b) = p(b \mid a) p(a)$$

• If $p(b \mid a)$ can be computed efficiently, represent only p(a) with samples and compute $p(b \mid a)$ for every sample Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

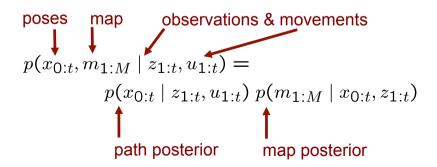


First introduced for SLAM by Murphy in 1999

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Rao-Blackwellization for SLAM

Factorization of the SLAM posterior



Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) = p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(m_{1:M} \mid x_{0:t}, z_{1:t})$$

How to compute this term efficiently?

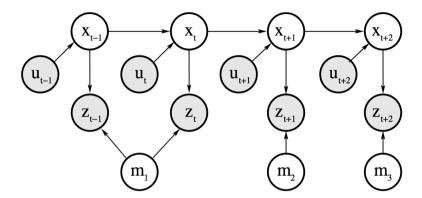
First introduced for SLAM by Murphy in 1999

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First introduced for SLAM by Murphy in 1999

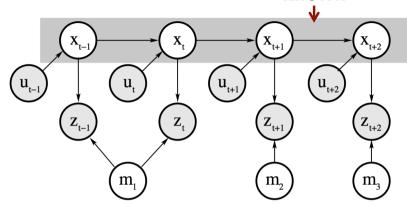
Revisit the Graphical Model



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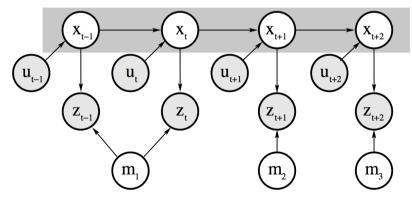
Revisit the Graphical Model

known



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Landmarks are Conditionally Independent Given the Poses



Landmark variables are all disconnected (i.e. independent) given the robot's path

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) = p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(m_{1:M} \mid x_{0:t}, z_{1:t})$$



Landmarks are conditionally independent given the poses

First exploited in FastSLAM by Montemerlo et al., 2002

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(m_{1:M} \mid x_{0:t}, z_{1:t})$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \prod_{i=1}^{M} p(m_i \mid x_{0:t}, z_{1:t})$$

First exploited in FastSLAM by Montemerlo et al., 2002

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Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

$$\begin{split} p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) &= \\ p(x_{0:t} \mid z_{1:t}, u_{1:t}) & p(m_{1:M} \mid x_{0:t}, z_{1:t}) \\ & \underbrace{\frac{p(x_{0:t} \mid z_{1:t}, u_{1:t})}{\text{\int}}}_{\text{particle filter similar to MCL}} p(m_{i} \mid x_{0:t}, z_{1:t}) \end{split}$$

2-dimensional EKFs!

First exploited in FastSLAM by Montemerlo et al., 2002

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Rao-Blackwellization for SLAM

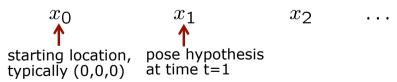
Factorization of the SLAM posterior

First exploited in FastSLAM by Montemerlo et al., 2002

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Modeling the Robot's Path

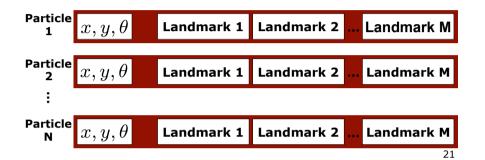
- Sample-based representation for $p(x_{0:t} \mid z_{1:t}, u_{1:t})$
- Each sample is a path hypothesis



- Past poses of a sample are not revised
- No need to maintain past poses in the sample set

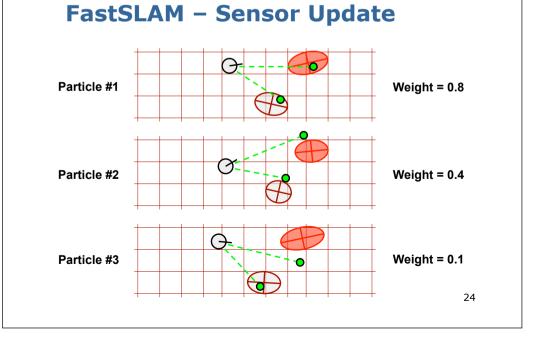
FastSLAM

- Proposed by Montemerlo et al. in 2002
- Each landmark is represented by a 2x2 EKF
- Each particle therefore has to maintain M individual EKFs

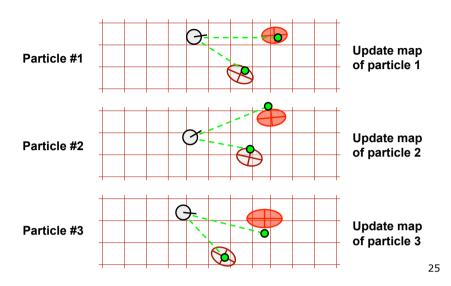


Particle #2 Particle #3 Landmark 1 2x2 EKF Landmark 2 2x2 EKF

Particle #2 Particle #3 Particle #3 Particle #3



FastSLAM - Sensor Update



Key Steps of FastSLAM 1.0

 Extend the path posterior by sampling a new pose for each sample

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

• Compute particle weight $w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \, \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} \, (z_t - \hat{z}^{[k]})\right\}$

measurement covariance

- Update belief of observed landmarks (EKF update rule)
- Resample

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FastSLAM 1.0 - Part 1

```
1: FastSLAM1.0_known_correspondence(z_t, c_t, u_t, \mathcal{X}_{t-1}):
2: for k = 1 to N do // loop over all particles
3: Let \left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle, \ldots \right\rangle be particle k in \mathcal{X}_{t-1}
4: x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t) // sample pose
```

FastSLAM 1.0 - Part 1

FastSLAM 1.0 - Part 2

```
11:
                          \langle \mu_{i,t}^{[k]}, \Sigma_{i,t}^{[k]} \rangle = EKF\text{-}Update(\dots) // update landmark
12:
                         w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}
13:
 measurement cov. \overset{\cdot}{Q}=H\;\Sigma_{i,t-1}^{[k]}\;H^T+Q_t exp. observation
                      endif
14:
                      for all unobserved features j' do
15:
                          \langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} \rangle
                                                                                      // leave unchanged
16:
17:
               end for
18:
              \mathcal{X}_t = \text{resample}\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)
20:
               return \mathcal{X}_t
```

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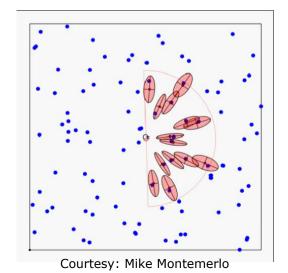
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FastSLAM 1.0 - Part 2 (long)

```
12:
                                                                             // measurement prediction
                                                                             // calculate Jacobian
<sup>14:</sup>update
                                                                             // measurement covariance
                                                                             // calculate Kalman gain
16:
                                                                             // update mean
17:
                                                                              // update covariance
18:
                                                 Q^{-1}(z_t - \hat{z}^{[k]}) // importance factor
19:
                   for all unobserved features j' do
                  \langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} \rangle endfor
21:
                                                                           // leave unchanged
23:
24:
             endfor
             \mathcal{X}_t = \text{resample}\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k-1}, N\right)
26:
             return \mathcal{X}_t
```

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FastSLAM in Action



The Weight is a Result From the Importance Sampling Principle

- Importance weight is given by the ratio of target and proposal in $x^{[k]}$
- See: importance sampling principle

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

The Importance Weight

The target distribution is

$$p(x_{1:t} \mid z_{1:t}, u_{1:t})$$

The proposal distribution is

$$p(x_{1:t} \mid z_{1:t-1}, u_{1:t})$$

Proposal is used step-by-step

$$p(x_{1:t} \mid z_{1:t-1}, u_{1:t}) = \underbrace{p(x_t \mid x_{t-1}, u_t)}_{\text{from } \mathcal{X}_{t-1} \text{ to } \bar{\mathcal{X}}_t} \underbrace{p(x_{1:t-1} \mid z_{1:t-1}, u_{1:t-1})}_{\mathcal{X}_{t-1}}$$

The Importance Weight

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) \ p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

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The Importance Weight

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_{t-1})}$$

Bayes rule + factorization

The Importance Weight

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_{t}^{[k]} \mid x_{t-1}, u_{t}) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta p(z_{t} \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_{t} \mid x_{t-1}^{[k]}, u_{t})}{p(x_{t}^{[k]} \mid x_{t-1}^{[k]}, u_{t})}$$

$$= \frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

The Importance Weight

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$= \frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

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The Importance Weight

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$= \frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

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The Importance Weight

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} = \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

$$= \eta \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j) \ p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dm_j$$

The Importance Weight

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} = \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

$$= \eta \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j) \ p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dm_j$$

$$= \eta \int p(z_t \mid x_t^{[k]}, m_j) \ p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1}) \ dm_j$$

The Importance Weight

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} = \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

$$= \eta \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j) \ p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dm_j$$

$$= \eta \int \underbrace{p(z_t \mid x_t^{[k]}, m_j)}_{\mathcal{N}(z_t; \hat{z}^{[k]}, Q_t)} \underbrace{p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(m_j; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} \ dm_j$$

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The Importance Weight

This leads to

$$w^{[k]} = \eta \int \underbrace{p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(m_j; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} \underbrace{p(z_t \mid x_t^{[k]}, m_j)}_{\mathcal{N}(z_t; \hat{z}^{[k]}, Q_t)} dm_j$$

$$Q = H \sum_{j,t-1}^{[k]} H^T + Q_t$$

$$w^{[k]} \simeq |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$$

The Importance Weight

This leads to

$$w^{[k]} = \eta \int \underbrace{p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(m_j; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} \underbrace{p(z_t \mid x_t^{[k]}, m_j)}_{\mathcal{N}(z_t; \hat{z}^{[k]}, Q_t)} dm_j$$

$$Q = H \sum_{j,t-1}^{[k]} H^T + Q_t$$

measurement covariance (pose uncertainty of the landmark estimate plus measurement noise)

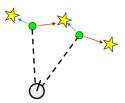
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FastSLAM 1.0 - Part 2

```
11:
                              \langle \mu_{i,t}^{[k]}, \Sigma_{i,t}^{[k]} \rangle = EKF\text{-}Update(\dots) // update landmark
12:
                             w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}
13:
                         endif
14:
15:
                         for all unobserved features i' do
                              \langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \tilde{\Sigma_{j',t-1}^{[k]}} \rangle \qquad // \text{ leave unchanged}
16:
17:
18:
                  endfor
                \mathcal{X}_t = \text{resample}\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)
19:
20:
                  return \mathcal{X}_t
```

Data Association Problem

• Which observation belongs to which landmark?

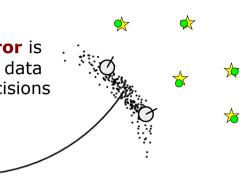


- More than one possible association
- Potential data associations depend on the pose of the robot

Particles Support for Multi-Hypotheses Data Association

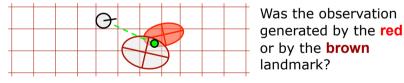
 Decisions on a perparticle basis

 Robot pose error is factored out of data association decisions



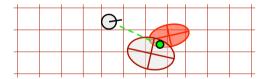
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Per-Particle Data Association



P(observation|red) = 0.3 P(observation|brown) = 0.7

Per-Particle Data Association



Was the observation generated by the **red** or by the **brown** landmark?

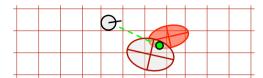
P(observation|red) = 0.3

P(observation|brown) = 0.7

- Two options for per-particle data association
 - Pick the most probable match
 - Pick an random association weighted by the observation likelihoods
- If the probability for an assignment is too low, generate a new landmark

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Per-Particle Data Association



Was the observation generated by the **red** or by the **brown** landmark?

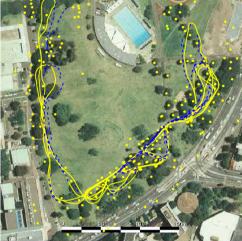
- Multi-modal belief
- Pose error is factored out of data association decisions
- Simple but effective data association
- Big advantage of FastSLAM over EKF

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Results - Victoria Park

- 4 km traverse
- < 2.5 m RMS position error
- 100 particles

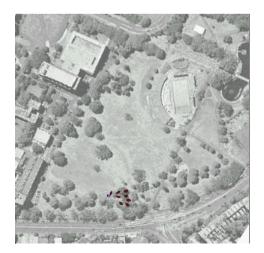


Blue = GPS Yellow = FastSLAM



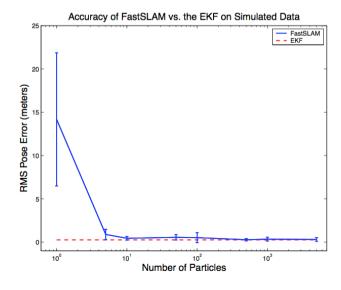
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Results – Victoria Park (Video)

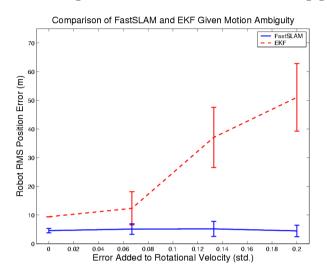


Courtesy: Mike Montemerlo

Results (Sample Size)



Results (Motion Uncertainty)



FastSLAM 1.0 Summary

- Use a particle filter to model the belief
- Factors the SLAM posterior into lowdimensional estimation problems
- Model only the robot's path by sampling
- Compute the landmarks given the path
- Per-particle data association
- No robot pose uncertainty in the perparticle data association

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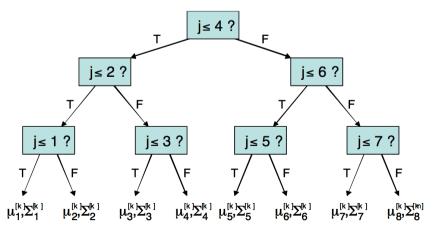
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FastSLAM Complexity – Simple Implementation

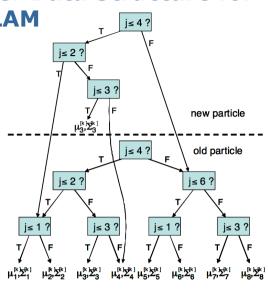
- Update robot particles $\mathcal{O}(N)$ based on the control
- Incorporate an observation $\mathcal{O}(N)$ into the Kalman filters
- ullet Resample particle set $\mathcal{O}(NM)$

N = Number of particles M = Number of map features $\mathcal{O}(NM)$

A Better Data Structure for FastSLAM



A Better Data Structure for FastSLAM



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FastSLAM Complexity

 Update robot particles based on the control $\mathcal{O}(N)$

• Incorporate an observation $\mathcal{O}(N\log M)$ into the Kalman filters

Resample particle set

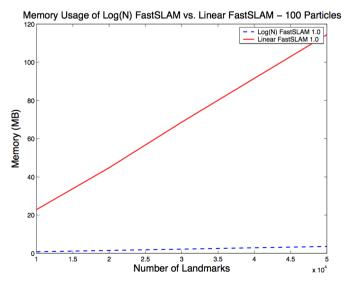
 $\mathcal{O}(N\log M)$

N = Number of particlesM = Number of map features

 $\overline{\mathcal{O}(N\log M)}$

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Memory Complexity



FastSLAM 1.0

 FastSLAM 1.0 uses the motion model as the proposal distribution

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

• Is there a better distribution to sample from?

FastSLAM 1.0 to FastSLAM 2.0

 FastSLAM 1.0 uses the motion model as the proposal distribution

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

- FastSLAM 2.0 considers also the measurements during sampling
- Especially useful if an accurate sensor is used (compared to the motion noise)

[Montemerlo et al., 2003] 61

FastSLAM 2.0 (Informally)

FastSLAM 2.0 samples from

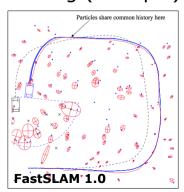
$$x_t^{[k]} \sim p(x_t \mid x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t})$$

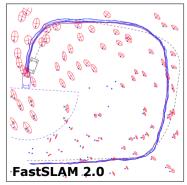
- Results in a more peaked proposal distribution
- Less particles are required
- More robust and accurate
- But more complex...

[Montemerlo et al., 2003] 62

FastSLAM Problems

- How to determine the sample size?
- Particle deprivation, especially when closing (multiple) loops





FastSLAM Summary

- Particle filter-based SLAM
- Rao-Blackwellization: model the robot's path by sampling and compute the landmarks given the poses
- Allow for per-particle data association
- FastSLAM 1.0 and 2.0 differ in the proposal distribution
- Complexity $\mathcal{O}(N \log M)$

FastSLAM Results

- Scales well (1 million+ features)
- Robust to ambiguities in the data association
- Advantages compared to the classical EKF approach (especially with nonlinearities)

Literature

FastSLAM

- Thrun et al.: "Probabilistic Robotics", Chapter 13.1-13.3 + 13.8 (see errata!)
- Montemerlo, Thrun, Kollar, Wegbreit: FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem, 2002
- Montemerlo and Thrun: Simultaneous Localization and Mapping with Unknown Data Association Using FastSLAM, 2003

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